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Complexity in Engineering and Natural Sciences
and
Centre for Nonlinear Studies

Overview of studies 1999-2015

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Abstract
An overview is presented on interdisciplinary studies into complexity of dynamical processes in the Centre for Nonlinear Studies (CENS) over years 1999-2015.

Contents

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Chapter 1
Introduction

This overview is about dynamics – forces and motions according to the definition but more specifically about waves propagating in continuous media. CENS has served as a basis for collaboration between various research groups studying waves in solids, water waves, dynamics in biological tissues, optics but also accompanying areas like control theory and proactive technology. In other words, this collaboration reflects the joint work of physicists, mathematicians, engineers and biologists. This explains also the title of the overview. CENS is certainly not alone with such ideas, there is a large community of researchers studying the complexity of the world and the research topics cover many more areas and fields. CENS is definitely a member of the international community and goes without saying that all the results of CENS are always embedded into the general framework of research worldwide.
The deep notions in contemporary science are nonlinearity and complexity. The studies on nonlinear effects in CENS were influenced by the earlier studies (1950s to 1960s) of Nikolai Alumäe (see, for example Engelbrecht and Kutser, 2015) who analysed the dynamical processes in thin shells with geometrical nonlinearities. Later on, the importance of nonlinearities became obvious in other areas (see Engelbrecht, 1999) and quite naturally the research turned to the path of complexity (Engelbrecht et al., 2010; Engelbrecht, 2015). Further these notions are described in more detail needed for better understanding of recent results.

Nonlinearity. In the nutshell, nonlinearity means a loss of proportionality. For a long time mankind followed the ideas of Leonardo da Vinci who set up in the 16th century his “rules of three” (see Truesdell, 1968). His second rule was about setting up the linear relations between variables under consideration. Although the gravitational law presented by Isaac Newton in the 17th century involved nonlinearity (inverse square dependence), it was only about a century ago when the scientific community accepted the deep meaning of nonlinearity. Indeed, as a result of nonlinear effects we nowadays understand are able to deal with shock waves, solitons, strange attractors, chaos, emergence of patterns, etc. In this context two aspects must be stressed. First as said by Maugin (1992) – it is not nonlinearity itself that influences the process but the “possibility of compensation/balance or competition between nonlinearity and another characteristic property”. This another characteristic property may be dispersion (the outcome is a soliton) or forcing (leading to chaotic regimes) or dissipation (leading to dissipative structures, etc. Second, the predictability of the process may be lost in nonlinear systems. The outcome is sensitive to the smallest changes in initial data. So, we may paraphrase Scheid (1993):

common sense usually so good at distinguishing between true and false fails in nonlinear science.
As explained by West (1985) and Engelbrecht (1997), in reality the normal world is nonlinear and sometimes the simplification to linear is possible, sometimes not. More on sources of nonlinearities and effects can be found in (Engelbrecht, 1997).

Complexity. Classical research aims to split-up general problems into their simpler components/constituents and then study them as deep as possible. But there is a threat like A.Toffler (1984) said:

One of the most highly developed skills in contemporary Western civilization is dissection: the split-up of problems into their smallest possible components. We are good at it. So good, we often forget to put the pieces back together again.

Nowadays we do put the pieces back into a whole and then we have to deal with complexity. The signatures of complexity in physical systems are described in many monographs (Nicolis and Nicolis, 2007; Érdi, 2008; etc). It means the following:
- complex systems are comprised by many different constituents which are connected in multiple ways;
- complex systems produce global effects including emergent structures generated by local interactions;
- complex systems are typically nonlinear;
- emergent structures occur far from equilibrium.

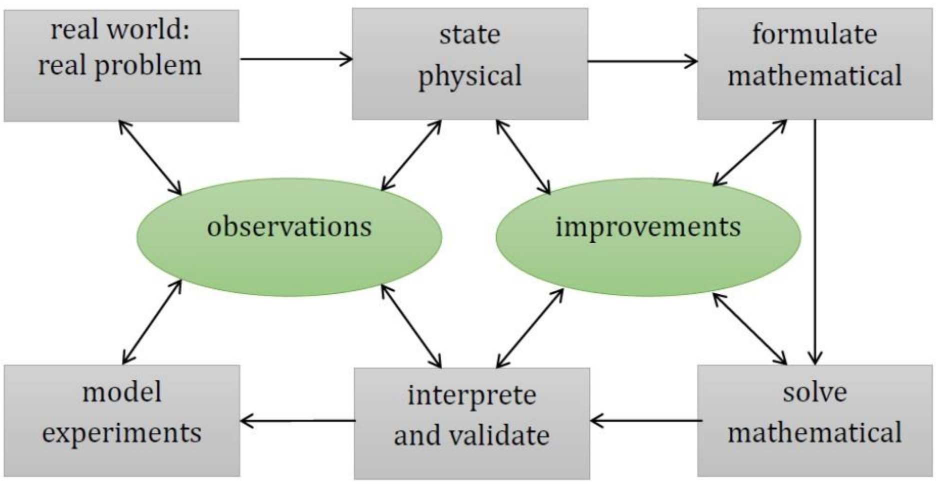
As a result we face:
- non-additivity and nonlinear interactions;
- deterministic unpredictability;
- sensitivity to initial conditions;
- typical phenomena like bifurcations, attractors, coherent states, multiple equilibria, etc;
- typical rules governing complex systems and methods for analysis.

Above is only a short-list of phenomena and methods in the nonlinear world. For more information one should consult the "Encyclopedia of Complexity and Systems Science" (Meyers, 2009).

Modelling. The main tool for understanding natural and artificial processes is mathematical modelling. This is not only a method for describing phenomena but it is also a tool for understanding phenomena.

The Book of Nature is written in the language of mathematic.
attributed to Galileo Galilei

One certainly cannot forget experiments but experiments do not explain explicitly the causality. The effective combination of mathematical modelling and experimental studies gives the best results. A possible flowchart of modelling and validation is shown in Figure below.



Models can be very complicated and can be very simple. It is again useful to remember famous thinkers:

Everything must be made as simple as possible but not simpler.
Albert Einstein

The modelling of dynamical processes in continuum media is based on well-elaborated background (see Maugin,2013). Excellent monographs by Truesdell and Toupin (1960), Eringen (1962), etc have built up a solid foundation. In studies of CENS the ideas of modelling have been described by Engelbrecht (1983; 1997; 2015). The conceptual approach starts from basic principles (initial assumptions and conservation laws) which must be complemented by constitutive theory (formulation of constitutive equations and auxiliary assumptions) resulting then in mathematical models (adding assumptions on field variables and constitutive equations). The conservation laws are fixed: conservation of mass, balance of momentum (the Newton's Second Law), balance of moment of momentum, conservation of energy, entropy inequality. Although these laws are fixed, on this basis an enormous number of various models can be built because the constitutive equations give many possibilities to model various real situations. However, there are axioms of constitutive theories (Eringen and Maugin, 1990) which must be satisfied: causality, determinism, equipresence, objectivity, time reversal, material invariance, admissibility, neighbourhood, and memory. A short description of these axioms is given also by Engelbrecht (2015). CENS has followed these principles throughout of studies. Dealing with biological processes, some special features should be taken into account like energy influx, chemical reactions, etc (Vendelin et al., 2007).

General remarks. Chris de Duve (Nobel awardee 1974) has analysed the unpredictability of scientific discoveries and pointed out two crucial questions:
- Do you know *where* are you going?
- Do you know *how* to get there?

If the answers to both questions are "yes" then it is applied research. If the answers are "yes" and "no", then it is either target-based (the first answer "yes") or knowledge-based (the second answer "yes") research, both reflecting different facets of basic research. If there are two "no's" to above questions, then it is serendipity. One could argue whether this last case exists nowadays or we have to leave it, for example to Sir Alexander Fleming.
Studies in CENS have mostly been characterized by one "no", usually it has been the answer to the second question. The reward is high – the richness of mathematical solutions and physical effects. However, one cannot deny several practical applications.

Summary. Despite several monographs authored by CENS members and several overviews, there is no general summary reflecting the basic philosophy of studies (briefly described above) and results obtained over all these years. What is collected now below is an attempt to generalize these results and to demonstrate how a bigger pattern is emerging from studies of dynamics within the framework of complexity.

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Chapter 2
Formulation of ideas

The period 1960-1990.

With the foundation of the Institute of Cybernetics at the Estonian Academy of Sciences in 1960, the basis for fundamental research in mechanics was created. The starting topics from this period involved the theory of thin shells (N.Alumäe) and photoelasticity (H.Aben).

Theory of shells. The main results were obtained in the theoretical analysis of the stability and vibrations of thin elastic shells and in the new research field at that time – the propagation of deformation waves in shells (see overview by Engelbrecht and Kutser, 2015). Thin shells are nonlinear and special new methods derived by N.Alumäe created for the analysis meant starting a new chapter in mechanics. In other words, the notions of nonlinearity and wave motion were introduced and these studies opened a wide avenue for future research. It must be stressed that the research was characterized by close cooperation with research centres in Moscow and Leningrad, although kept behind the Iron Curtain of this time. The creative atmosphere in the Institute of Cybernetics has attracted many researchers to join the research group. L. Ainola has formulated variational principles for dynamical problems where a novel idea of convolution integrals has been used. Such an approach has permitted to build up the nonlinear and linear models of shell theories (Ainola, 1967). The qualitative and numerical studies of axisymmetric and plane transient stress waves have demonstrated the domains with qualitatively different stress states which permitted to determine the effectiveness of different shell theories (Nigul, 1969).

These studies have generated two main flows of research – (i) studies of echosignals from elastic bodies (shells) under acoustical impact and (ii) the analysis of wave phenomena in continuous media with a special attention to nonlinearity.

In the first direction the attention was devoted to the analysis of diffraction of acoustical signals on elastic objects, mainly on elastic spherical and cylindrical shells. The mechanisms of formulation of echo-signals from such objects were determined and their informative character used for building the algorithms of solving the inverse problems (Metsaveer et al., 1979; Veksler, 1989). The roles of Lamb waves, Rayleigh waves and Whispering Gallery waves have been established and the informative indicators of echo-signals have been determined. The echo-signals from layered objects were used by Nigul (1981) for solving the inverse problem taking into account the nonlinear effects.

In the second direction, the basis was laid by building of Lagrangian mathematical models for deformation waves in thermoelastic media (Nigul and Engelbrecht, 1972) which later has been generalized for general mathematical theory for deformation waves (Engelbrecht and Nigul, 1981). A detailed analysis of nonlinear wave deformation processes described the derivation of evolution equations and several physical effects in thermoelastic and viscoelastic media (Engelbrecht, 1983). Several methods used for deriving the evolution equations were generalized into a unified framework (Pelinovski et al., 1984). Wave processes in hereditary materials were studied (Nigul, 1989; Ravasoo, 1989) bearing in mind the inverse problems of determining the material characteristics from measurements (acoustodiagnostics). Several cases were analysed: (i) homogeneous nonlinear elastic solids with inhomogeneous predeformation; (ii) homogeneous viscoelastic solids with nonlinear instantaneous elastic response and linear viscoelastic response; (iii) inhomogeneous viscoelastic solids with space-dependent properties. A mathematical model for description of longitudinal or transverse waves with possible amplification in the Earth's crust has been proposed Engelbrecht and Khamidullin, 1988). In this case the governing equation is the celebrated Korteweg- de Vries equation with a forcing term which takes into account the block structure of the crust.

Photoelasticity. Another important direction of studies from 1960 on was the theory and application of photoelasticity under the supervision of H.Aben (1979). The photoelasticity is based on the coupling of light field and the deformation field in transparent objects. First, the method of characteristic directions has been elaborated and effectively used in many applications (stress analysis in shells, plates, and twisted glass fibres). Second, the ideas of integrated photoelasticity (optical tomography) have been used for determining the space-dependent stress fields. This method belongs actually to the class of hybrid mechanics because beside the optical measurements, the theory of elasticity has been used. It was shown how in some cases the tensor tomography is reduced to the case of scalar tomography. The stress states of cubic single monocrystals have been determined. Third, the novel method of magneto-photoelasticity based on the coupling of light and magnetic fields has been proposed and realized in practice.

Other studies. Several researchers who joined the Institute of Cybernetics (and CENS) later have prepared the ground for further studies in various other centres. Their topics were actually close to problems described above. For example, the convective transport in magnetic fields has revealed the generation of small scales (Kalda and Kingsep, 1989). It has been shown that a magnetohydrodynamic wave breaks in a magnetized plasma (Isichenko and Kalda, 1989). An interesting problem of modelling the Rossby waves in ocean has been studied by Soomere (1987), the kinetic equations were derived, the energy spectrum established, the nonlinear synoptic motions in ocean analysed (Soomere and Strachuk, 1990), etc. The vortices in viscous fluids explain special motions in fluids which are needed for understanding turbulence (Berezovski, Kaplanski, 1989), for example, the buoyancy effect.

From all these studies the keywords started to emerge: nonlinearities, fields, and coupling effects. The continuous media under consideration could be solid or fluid or plasma. The general character of wave processes opened the eyes for neighbouring fields like dynamical processes in biological tissues. For example, the nerve pulse propagation could be modelled using the ideas from the theory of evolution equations (Engelbrecht, 1981). The ideas of acoustodiagnostics could be used also for describing ultrasonic waves in soft tissues (Engelbrecht and Chivers, 1989).

The time was ripe for further consolidation and focusing of ideas like described in Chapter 1.

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The period 1991-1998.

During this period the studies on nonlinear dynamics were intensified due to the consolidation of topics supported by reinstating the independence of Estonia. Below the main results are described based on continuation of earlier studies together with new problems like fractality in natural processes and biophysical problems.

Theoretical modelling. Beside using the clear modelling principles of elastic continua (Engelbrecht, 1983), the attention was turned to thermodynamical problems. The idea was to analyse simple subsystems in the compound of a complex system. The formalism of based on the compatibility of between the thermodynamical laws and equations of state is proposed and equilibrium conditions for complex systems formulated (Berezovski, Rosenblum, 1993a). This led to the possible structuring which is based to the interchange of energy between constituents of a complex thermodynamical system (Berezovski, Rosenblum 1993b). An algorithm based on the idea of continuous cellular automata was developed for the simulation of 1D heat conduction and thermoelasticity (Berezovski, 1997) and several problems solved: waves and shocks in thermoelastic homogeneous and layered media, heat conduction in regions of phase transitions, etc. In physical terms, it was explained how the energy of a complex system can be described in terms of energies of its constituents and the interactions between them. The contact quantities were introduced for describing the non-equilibrium states of constituents (Berezovski, Maugin, 1998a, 1998b) which turned to be extremely effective to increasing the accuracy of numerical methods.

The concept of internal variables has been used for thermodynamical presentation of nerve pulse models (Maugin, Engelbrecht, 1994) and for modelling deformation waves in thermoelastic media (Engelbrecht, Maugin, 1996).

For further studies one should stress the notion of internal variables and the importance of contact quantities in thermodynamical systems.

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Nonlinear wave propagation. Solitary waves form a paradigm in contemporary mathematical physics. A starting point to further studies was summarized in the book by J.Engelbrecht (1991) devoted to the analysis of asymmetric solitary waves. The asymmetry is caused by the energy influx to the system. Two different types of such asymmetric waves are described: those for which the energy change is of a perturbative character (perturbed Korteweg-de Vries (KdV) type) and those for which the energy change is the essential feature of their existence (nerve pulse equations). A large cycle of studies is devoted to the analysis of the KdV type equations. For a standard KdV equation the systematic spectral analysis has given new information about the spectral properties of emerging KdV solitons over a large range of dispersion parameters (Salupere et al., 1994). This was possible by using the pseudospectral method for numerical simulation with accuracy much higher than the conventional methods could give. The number of emerging solitons is estimated and hidden (virtual) solitons detected from the interaction analysis (Salupere et al., 1996).
The governing equation for waves in austenitic-martensitic alloys is the KdV-like equation with the quartic nonlinearity and the third and fifth order dispersion (Salupere et al,1997). The solutions of such an equation can be either usual N-soliton trains, or dark N-soliton trains or chaotic solutions.For a forced KdV equation with a polynomial forcing term, the regions where solitons are amplified or suppressed are found and the types of corresponding solitonic structures (single soliton, double soliton, cnoidal wave , etc) estimated (Peterson, Salupere, 1997). The waves in shallow water are described by the Kadomtsev-Petviasvili equation. The solution to this equation are found in terms of Hirota bilinear formalism (Peterson, 1998). The inverse problem for two-soliton and periodic two-soliton solutions is stated and solved, i.e. finding the amplitudes of waves using only a snapshot of a wave pattern. As a result, the notion of interaction soliton is introduced which exist only for a certain short period (Peterson, 1998).
A seminal overview was published by Engelbrecht and Braun (1998) where the influence of two essential qualities of solids – nonlinearity and nonlocality on wave motion were analyzed. As far as nonlocality is related to the microstructure of solids, these preliminary ideas paved the way to further studies of the CENS.
A summary of all results concerning the studies of wave motion was presented by a monograph by J.Engelbrecht (1997). The general idea advocated in this monograph is to start from complicated mathematical models of continua and then to find a simpler viewpoint

preserving still everything essential. The selected examples showed some unconventional approaches in order to demonstrate the richness of nonlinear wave motion. All examples were cast into a general framework of complexity and simplicity – again an approach which was later used in the CENS. The essays on nonlinearity, beauty and complexity demonstrated the viewpoints usually shadowed by mathematics.

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Impact and piano hammers. The sound generation in pianos depends on properties of piano hammers made of felt. A new hysteretic model of the grand piano hammer felt was proposed (Stulov, 1993;1995). In this model, the felt was assumed as a nonlinear hereditary material with an exponential relaxation kernel describing both the loading and unloading stages of an impact. This is important for the analysis string-hammer interaction as the main mechanism of generating the sound. The mensurable sets of strings were calculated for pianos Estonia-Mignon and Baby-Grand and also for the medium-sized Grand Piano. These sets were characterized by the unbroken form of the tension distribution per choir which increased the quality of the instruments (Stulov, 1997). The studies related to concrete pianos were obtained in the close cooperation of the Tallinn Piano Factory. A new device was constructed for measuring the force-deformation characteristics of hammers during the impact.

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Integrated photoelasticity. The studies in photoelasticity continued within the framework of tensor tomography (Aben and Guillemet, 1993). The earlier elaborated method of characteristic directions is actually nonlinear due to nonlinear relations between the parameters of the stress field and measured quantities (Ainola and Aben, 1999). This nonlinear effect leads to the appearance of interference plots and fringe dislocations (Aben and Josepson, 1997; Aben and Ainola, 1998). The methods proposed for the stress analysis had high accuracy (Aben Josepson, 1995) and were used for determining stress fields in specimens of complicated shape like tempered tumblers, optical fibres, bottles, cathode ray tubes, etc. It has been shown that the method of integrated photoelasticity allowed also to determine nonaxisymmetric residual stresses in axisymmetric objects. Many such applications were realized by international contracts. It was also possible to account for thermal stresses (Aben and Ainola, 1996). In order to carry out measurements with high accuracy, the automatic polariscopes were designed together with the needed software (Aben et al, 1999). These polariscopes are used in many factories over all the world (Asahi Funabashi, 1995; Arc International, 1996, Philips, 1998).

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Acoustodiagnostics. Theoretical studies were focused to elaboration of physically well-grounded algorithms for non-destructive evaluation (NDE) of material properties or stress states by acoustical signals. It needed clear distinction of homogeneous and inhomogeneous predeformation cases and well-defined constitutive equations. The derivation of constitutive equations of nonlinear viscoelastic media (Ravasoo, 1991) paved the way to extend the possibilities of the NDE by using, beside the wave velocity measurement data, also data about the wave profile evolution (Ravasoo,1993). For this, the special software was generated. The dependences of wave characteristics on the parameters of predeformation were determined which enabled to propose an algorithm for the NDE of the inhomogeneous predeformed state of a medium (Ravasoo, 1995; 1999). A theoretical model of nonlinear viscoelasticity was checked against experimental data and a modified constitutive equation for nylon threads was proposed (Blanc and Ravasoo, 1996). A method and an algorithm for the NDE of an inhomogeneous material under inhomogeneous prestress were derived using nonlinear effects by wave interactions (Ravasoo and Braunbrück, 1999) which may lead to amplification of a signal.

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Scattering problems. For a certain period, the studies on wave scattering by elastic objects were going on before researchers left the Institute. The overview on earlier results on the resonance scattering was published as a monograph (Veksler, 1993). From studies of this period some should be mentioned. It was shown that the resonance components of peripheral waves generated by a plane acoustic wave obliquely incident on cylindrical shell of infinite extent include symmetric and asymmetric Lamb waves (Veksler, 1993). It was shown that for a plane wave scattering by acoustically rigid and soft spheres and cylinders the influence of Franz waves is significally different for rigid and soft cases (Veksler and Izbicki, 1995). A review on the application of the Resonance Scattering Theory is presented (Veksler, 1994). From 1996 on, these studies have been moved to the University of Le Havre.

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Biophysics – cardiac processes. Based on the earlier results, the cardiac arrhythmias were studied by methods of contemporary nonlinear dynamics. The conduction was modelled by the nonlinear nerve pulse equation (NPE) which is an evolution equation for one wave. (see Engelbrecht, 1991). An important result was the detection of the non-entrant bistability of cardiac Purkinje cells which were mostly affected by the driving conditions and the level of its supernormality. Such a bistability on the cell level can induce the bistability on the tissue level leading to the bistability of the cardiac conducting system during tachyarrhythmias. Mathematical models were elaborated (Engelbrecht and Kongas, 1993; 1995) and the possible bifurcation types of oscillations established (Kongas, 1998; Kongas et al., 1999). Using the NPE as a model, it was shown that the threshold property with a supernormal excitability of a tissue were sufficient conditions for generating Wenckebach- and Mobitz2-type arrhythmias (Kongas and von Hertzen, 1996). The role of co-existing attractors in arrhythmias is demonstrated.
Beside the propagation of electrical signals, the energy fluxes play an important role in intracellular diffusion and energy consumption in heart muscles. A mathematical model of compartmentalized energy fluxes was developed combining the model of intracellular energy transduction and the model of oxidative phosphorylation (Saks et al. 1998). These studies supported building a mathematical model for heart contraction including the active stress development and energy consumption of the heart muscle (Vendelin et al. 1999). The studies were supported by active international cooperation (Grenoble, Eindhoven).

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Biophysics – fractality. Beside the cardiac phenomena, the tree-like fractal biological networks were studied. A fractal model of the human blood-vessel system was proposed as a generalized Scheiddeger's model of rivers and its fractal properties determined (Kalda, 1993; 1998) including the concept of overlapping exponent which characterizes the self-similar systems. Transport processes in biological fractal structures have been analysed and the governing scaling laws established (Kalda, 1999). Applying a new technique of calculating the similarity dimension, the fractal dimensions of 3D objects from their 2D images (like a photo of a tree) were determined. The propagation of infection in a fractal-like structure of lung was analyzed (Kalda 1996). The calculations of fractal dimensions were optimized in Monte-Carlo simulations (Kalda 1997).
A method for analysis of heart rate variability using the theory of fractals was proposed based on data from Holter-monitoring. A new Zipf-law-based multiscaling behaviour of the heart rate variability was discovered (Kalda et al., 1998).

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Processes in nature. Multifractality of the Estonian coastline was analyzed and its fractal dimensions calculated. It was shown that the more involved parts of the coast were exposed to the influence of storms at a lesser extent, so that the smaller structures were less smoothened (Kalda, 1992). The fractal dimensions of coastlines were studied also in the general context of statistical topography (Isichenko and Kalda, 1991). It was shown that based on the statistical topography of the stream function it is possible to model the propagation of a passive tracer in a 2D turbulent flow (Kalda, 1994).

The studies in neighbouring research groups must also be mentioned in view of the further formation of CENS. The kinetic equations governing the Rossby waves were solved and the resonance conditions established (Soomere and Rannat, 1991; Soomere, 1993). It led to general problems of geostrophic turbulence (Soomere, 1995) with the spectral analysis (Soomere, 1996).

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Summary. From this brief analysis it is obvious that many elements of complexity are seen from studies: constituents of processes, interaction of waves and fields, nonlinearity, patterns of dynamical processes, etc. So the time was ripe to focus all the studies related to these fields of knowledge under one umbrella. This was done in 1999 when the Centre for Nonlinear Studies (CENS) was founded following the ideas presented in Chapter 1. Certainly CENS was built on the strength of former studies with the clear idea to move on to new problems. This was very much along the lines set up by N.Alumäe, the founding director of the Institute of Cybernetics in 1960.

From the studies up to 1999, several keywords are taken along to CENS: waves, nonlinearity, solitons, internal variables, biosystems, fractality, turbulence, thermodynamics. Most important was the understanding that many neighbouring fields of research enrich each other and the views of interdisciplinarity open new avenues of research.

Next Chapters describe the results by various fields. The philosophy of CENS is described in Annex 1, the groups of CENS listed in Annex 2 and the members of the International Advisory Board – in Annex 3, etc.

Chapter 3
Waves in solids

Preliminaries
The cycle of studies into wave propagation in solids was based on the mathematical modelling of physical phenomena attributed to waves. As stated in Chapter 1 (Introduction), the modelling has several stages and the conceptual approach is well understood (see references to Chapter 1). Based on earlier studies over 1991-1998 (Chapter 2), the attention was focused to microstructured solids. These are solids which have clearly distinguishable small scale internal structure at the mesoscale which has an effect on the macroscale processes. Such solids (materials) are, for example, the functionally graded materials (FGMs), granular solids, ceramics, polycrystalline alloys, fibre reinforced solids, etc. In addition the studies involved also waves in solids with discrete microstructures.
The main sequence of studies for every particular case was as follows: (i) well-grounded constitutive theory; (ii) derivation of motivated mathematical models – wave equations; (iii) solving and analysing the mathematical problems; (iv) analysing the physical effects. In CENS, there were no possibilities for physical experiments and therefore the results were checked, if possible, against experiments carried on in other research centres.

Modelling
(i) First, the micromorphic theory of Mindlin (1964) has been used as basis for the mathematical modelling of waves in microstructured solids. It has been shown that the Euler-Lagrange equations form a suitable framework for deriving the governing equations for waves (Engelbrecht et al., 2005). In this case, the kinetic K and potential W energies included macrogradients (u_x and u_t) and microdeformation and its gradients (φ, φ_t, and φ_x):

$$K = K(u_t, \varphi_t), \quad W = W(u_x, \varphi, \varphi_x). \tag{3.1}$$

Then the governing equations (two balance equations for macro- and microstructure, respectively) were easily derived (see, for example Engelbrecht et al., 2005; Janno and Engelbrecht, 2011; Engelbrecht, 2015) involving also the inertia of the microstructure. Further analysis (see below) showed that this is an important aspect in explaining physical effects in microstructured solids. A generalization of such an approach permitted to model also wave processes in solids with two concurrent or two hierarchical (a scale within a scale) microstructures and to derive the corresponding governing equations (Engelbrecht et al, 2007). The derived governing equations included also the physical nonlinearities at the macro- and micro-scale.

(ii) Second, an essential generalization followed which had many consequences. Namely, in order to model microstructure as a field, the concept of dual internal variables was introduced (Ván et al, 2008), which brought thermodynamics directly into modelling of waves. Internal variables are supposed to describe the behaviour of the unknown for an external observer internal structure and in this sense compensate our inability to precicely describe the microstructure characteristics. The conventional formalism of internal variables is used for the description of dissipative effects (Maugin, 2015) and leads as a rule, to the first-order reaction-diffusion equations. We have proposed a modified formalism which is sufficiently general to comprise the micromorphic elasticity and thermoelasticity in addition to classical applications. Based on canonical balance equations for material momentum and energy

(Maugin, 2006), the governing equations are derived. According to this approach, the balance of momentum is used

$$\left.\frac{\partial \boldsymbol{P}}{\partial t}\right|_{x} - Div_{R}\boldsymbol{b} = \boldsymbol{f}^{int} + \boldsymbol{f}^{ext} + \boldsymbol{f}^{inh} \tag{3.2}$$

where $\boldsymbol{P}$ is the material momentum, $\boldsymbol{b}$ is the material Eshelby stress and $\boldsymbol{f}^{int}, \boldsymbol{f}^{ext}, \boldsymbol{f}^{inh}$ are the material inhomogeneity force, the material external force and the material internal force, respectively.

The potential energy is taken in the form:

$$W = W(\boldsymbol{F}, \theta, \boldsymbol{\alpha}, \nabla_{R}\boldsymbol{\alpha}, \boldsymbol{\beta}, \nabla_{R}\boldsymbol{\beta}), \tag{3.3}$$

where $\boldsymbol{F}$ is the macrodeformation gradient, θ is the temperature and $\boldsymbol{\alpha}, \boldsymbol{\beta}$ are internal variables, each of which is a second-order tensor.

The dissipation inequality reads:

$$\phi = \phi(\dot{\boldsymbol{\alpha}}, \dot{\boldsymbol{\beta}}, \nabla_{R}\theta) \geq 0. \tag{3.4}$$

The governing equations for the internal variables are determined from the dissipation inequality

$$\begin{pmatrix} \dot{\boldsymbol{\alpha}} \\ \dot{\boldsymbol{\beta}} \end{pmatrix} = R \begin{pmatrix} \tilde{\mathcal{A}} \\ \tilde{\mathcal{B}} \end{pmatrix}, \tag{3.5}$$

where linear operator **R** depends on state variables and

$$\tilde{\mathcal{A}} = -\left(\frac{\partial W}{\partial \boldsymbol{\alpha}} - \mathrm{Div}_{R}\frac{\partial W}{\partial(\nabla_{R}\boldsymbol{\alpha})}\right), \Phi \tag{3.6}$$

$$\tilde{\mathcal{B}} = -\left(\frac{\partial W}{\partial \boldsymbol{\beta}} - \mathrm{Div}_{R}\frac{\partial W}{\partial(\nabla_{R}\boldsymbol{\beta})}\right). \tag{3.7}$$

These equations are not limited to reaction-diffusion equations but may include hyperbolic governing equations which describe also inertia of the microstructure and interaction of waves at macro- and micro-levels. The dual internal variable approach offers a unified description of dissipative and non-dissipative internal processes in solids within the framework of continuum mechanics. Summing up, in contrast to other theories the following should be stressed (Engelbrecht, 2015):

- One balance equation of momentum and dissipation equation are used instead of two balance equations in the micromorphic theory.
- Governing equations for internal variables follow from the dissipation inequality and are therefore thermodynamically consistent.
- Governing equations for internal variables are not restricted to first-order equations and may include also second-order derivatives responsible for wave motion.
- Boundary conditions for internal variables are determined by zero extra entropy flux at a boundary, which is a natural condition for internal variables.

The detailed description of the theory and applications of the dual internal variables is given by Berezovski et al. (2011a, 2011b); and Engelbrecht and Berezovski (2012). It has been demonstrated that the structure of Cosserat, micromorphic and second gradient theories can be recovered in terms of dual internal variables in a natural way (Ván et al, 2014). The comparison with other theories of microstructured solids is presented by Engelbrecht and Berezovski (2015).

As an independent application, the concept of internal variables has also been used for describing the cardiac muscle contraction (Engelbrecht et al., 2000). In this case, a specific feature was the hierarchical structure of internal variables.

(iii) Third, the microstructure can be aligned in a material and in this case the microscopic directors must be taken into account. For example, this is the case of fibre-reinforced materials (concrete, composites). For modelling such solids, the concept of alignment tensors was introduced (Herrmann, Eik, 2014; Eik et al., 2015). The strain energy density function is then composed by two parts: the isotropic part describing concrete (matrix) and the orthotropic part describing fibres and depending on their orientation parameters. For determining the fibre orientation distribution parameters, a special virtual reality visualization environment was built and corresponding software developed (Pastorelli, Herrmann, 2013).
In case of wool felt used in piano hammers, the Cauchy's method was applied for deriving the governing equations. A special device was constructed for measuring the firce-compression characteristics of different hammers (Stulov, Mägi, 2000) with a clear difference in loading and unloading regimes. On the basis of those experiments, a hereditary model for stress-strain relation $(\sigma = \sigma(\varepsilon))$ was proposed (Stulov, 2004):

$$\sigma(t) = E_d \left[\varepsilon^P(t) - \frac{\gamma}{\tau_0} \int_0^t \varepsilon^P(\tau) \exp\left(\frac{t - \tau}{\tau_0} \right) d\tau \right], \tag{3.8}$$

where E_d is the dynamic modulus, $p \geq 1$ is a material parameter (compliance nonlinearity exponent, γ is the hereditary amplitude and τ_0 is the relaxation time. The parameters were determined from experiments.

(iv) Fourth, several special problems of continuum theory were analysed related to the appearance or to the existence of internal boundaries in solids. One cycle of studies concerned the modelling of discontinuities in solids that could appear in phase transitions or in propagating cracks. A thermomechanical description of coherent martensitic phase-transition fronts was developed based on the material description. This description needs the derivation of stability conditions which were formulated by means of contact quantities. The driving traction f_S acting on the phase boundary was described by a kinetic relation and the criterion for a nucleation of an austenitic-martensitic phase transition was formulated (Berezovski, Maugin, 2002; 2003). The concept of the driving traction and the corresponding kinetic relation was also applied for describing the Mode I brittle cracks (Berezovski, Maugin, 2005; 2007). In this case, the driving force is proportional to the energy release rate at the crack tip (Berezovski, Maugin, 2005). A detailed overview of these studies is presented by Berezovski et al. (2008).
Another cycle of studies concerned microstructured solids which have a regular structure like layers and gratings or isolated inclusions. For the analysis of waves in such solids the numerical methods have proved to be most effective tools. Here the earlier theoretical considerations applied for the analysis of thermodynamical systems (see Chapter 2) have been extremely useful. Namely, the construction of an appropriate numerical algorithm involved the calculation of contact (excess) quantities at boundaries which guaranteed the thermodynamical consistency. For modelling the discrete structures, the linear and nonlinear theory of elasticity has been used (Berezovski et al., 2008). In modelling of scattering problems, contrary to the conventional approach, also the elasticity of scatterers (grating) was taken into account (Berezovski et al., 2015).

(v) Finally, one cycle of studies was related to continuum physics. The orientation of liquid crystals in the mesoscopic scale can also be treated as a microstructure. Contrary to the standard formulation it was proposed that the balance of spin is a component equation of the balance of momentum. That gave a possibility to reformulate the theory of mesoscopic continuum physics (Herrmann, 2009). The stress tensor includes then also rotational and mixed spatial-rotational parts responsible for coupling of the coupled stress with other stress components. As a consequence, the derivation of a wave equation for twist waves is simplified (Herrmann 2009; Herrmann, Engelbrecht, 2010). Such an analysis has revealed also some open questions in mesoscopic continuum physics. Namely, the problem of (virtually) disconnected mesoscopic domains has been discussed. It was shown that continuous three-dimensional domains may become discontinuous and then strongly non-local formulation of constitutive function is required because a weakly non-local formulation (including gradients) is not sufficient anymore (Herrmann, Engelbrecht, 2012).

Summing up. The studies into continuum theory have shown that it is possible to elaborate well-grounded and thermodynamically consistent theories accounting for different microstructures. For this purpose special concepts have been introduced like dual internal variables, contact quantities, alignement tensors, etc.

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Wave equations

The next stage of mathematical modelling needs solvable governing equations. For this purpose, the wave equations were derived from the constitutive theories. Most of derived equations presented below are one-dimensional that gives a transparent view on essential physical effects and possible couplings. Beside the straightforward two-wave equations, the one-wave evolution equations are presented. The representative selection of equations reflects those which were studied in more detail because of their physical importance.

(i) Two-wave equations.

The balance laws for microstructured solids with uniform distribution of the microstructure take the form (Engelbrecht et al., 2005; Engelbrecht, 2015) in the linear setting:

$$\rho u_{tt} = (\lambda + 2\mu)u_{xx} + A\varphi_{x,} \tag{3.9}$$

$$I\varphi_{tt} = C\varphi_{xx} - Au_x - B\varphi, \tag{3.10}$$

where u is the macrodisplacement, φ is the microdeformation, ρ is the macrodensity, λ, μ are the Lamé parameters, I is the microinertia, and A, B are material parameters. The Eqs. (3.9), (3.10) are derived either by using Euler-Lagrange equations (Engelbrecht et al., 2005) or by using the concept of dual internal variables (Berezovski et al., 2011). It is possible to represent system (3.9), (3.10) in the form of one equation:

$$u_{tt} = (c_0^2 - c_A^2)u_{xx} - p^2(u_{tt} - c_0^2 u_{xx})_{tt} + \\ + p^2 c_1^2 (u_{tt} - c_0^2 u_{xx})_{xx} = 0, \tag{3.11}$$

where

$$c_0^2 = (\lambda + 2\mu)/\rho_0, \quad c_1^2 = C/I, \quad c_A^2 = A^2/\rho, \quad p^2 = I/B. \tag{3.12}$$

Using the scaling principle, Eq. (3.11) yields

$$u_{tt} = (c_0^2 - c_A^2)u_{xx} - p^2 c_A^2 (u_{tt} - c_1^2 u_{xx})_{xx}\,. \tag{3.13}$$

This equation demonstrates clearly the hierarchical structure of the wave process.
In nonlinear setting instead of system (3.9), (3.10) one obtain (Engelbrecht et al., 2007)

$$\rho u_{tt} = (\lambda + 2\mu)u_{xx} + A\varphi_x + N u_x u_{xx}, \tag{3.14}$$

$$I\varphi_{tt} = C\varphi_{xx} - Au_x - B\varphi + M\varphi_x\varphi_{xx}, \tag{3.15}$$

where *N* and *M* are the nonlinear constants at macro- and microlevel, respectively.
If microstructure is space-dependent then system (3.9), (3.19) is modified (Engelbrecht, Berezovski, 2015):

$$\rho u_{tt} = (\lambda + 2\mu)u_{xx} + A\varphi_x + A_x\varphi, \tag{3.16}$$

$$I\varphi_{tt} = C\varphi_{xx} - Au_x - B\varphi + C_x\varphi_x, \tag{3.17}$$

where gradients $A_{x,}$ C_x are accounted for.
Another possibility to derive a governing equation for space-dependent properties of a solid is to assume all the material constants be space-dependent. Then the wave equation can directly by derived from the balance of momentum (Ravasoo, Braunbrück, 2005; Ravasoo, 2014):

$$k_4(x)u_{tt} - [1 + k_1(x)]u_{xx} - k_2(x)u_x - k_3(x)u_x^2 = 0, \tag{3.18}$$

where $k_1, k_2, k_{3,}$ $k_{4,}$ are space-dependent material functions.
In the presence of two microstructures with microdeformations φ_1 and φ_2 (φ_2 within φ_1) the governing equations are (Engelbrecht et al., 2007)

$$\rho u_{tt} = (\lambda + 2\mu)u_{xx} + A_1(\varphi_1)_x, \tag{3.19}$$

$$I_1(\varphi_1)_{tt} = C_1(\varphi_1)_{xx} - A_1u_x - B_1\varphi_1 + A_{12}(\varphi_2)_x, \tag{3.20}$$

$$I_2(\varphi_2)_{tt} = C_2(\varphi_2)_{xx} - A_{12}(\varphi_1)_x - B_2\varphi_2, \tag{3.21}$$

where I_1 and I_2 are the corresponding microinertia and the coefficients A_1, B_1, B_2, A_{12} reflect coupling effects.
System (3.19) – (3.21) can be represented by one equation (Engelbrecht et al., 2007)

$$\begin{aligned} u_{tt} - (c_0^2 - c_A^2)u_{xx} = p_1 c_{A1}^2 [u_{tt} - (c_1^2 - c_{A2}^2)u_{xx}]_{xx} - \\ - p_2^2 c_{A1}^2 c_{A2}^2 (u_{tt} - c_2^2 u_{xx})_{xxxx}, \end{aligned} \tag{3.22}$$

where

$$\begin{aligned} &c_{A1}^2 = A_1^2/\rho B_1, \quad c_{A2}^2 = A_2^2/\rho B_2; \quad c_1^2 = C_1/I_1, \quad c_2^2 = C_2/I_2, \\ &p_1^2 = I_1/B_1, \quad p_2^2 = I_2/B_2\,. \end{aligned} \tag{3.23}$$

Again, the hierarchical structure of Eq. (3.22) reflects the hierarchy of the wave process. The governing equations for concurrent microstructures φ_1 and φ_2 are in principle similar to Eqs. (3.19) – (3.21) with different coupling terms (Berezovski et al., 2010).
In case of the wool felt with the stress-strain relation (3.8), the governing equation reads (Kartofelev, Stulov, 2014)

$$[(u_x)^p]_x - u_{tt} + [(u_x)^p]_{xt} - \delta u_{ttt} = 0. \tag{3.24}$$

For a biomembrane, the density change $\Delta\rho = u$ along the biomembrane; the governing equation takes the form (Heimburg, Jackson, 2005; Engelbrecht et al., 2015):

$$u_{tt} = [(c_0^2 + pu + qu^2)u_x]_x - h_1 u_{xxxx} + h_2 u_{xxtt}, \tag{3.25}$$

where p, q, h_1 and h_2 are material constants charscterizing the lipid structure of a biomembrane. Here the nonlinearity is expressed by u-type terms contrary to convential solids where the nonlinearity is of u_x-type.

(ii) One-wave (evolution) equations. The evolution equations describe only either to the right or to the left propagating waves. The procedures how to extract the wave equations from the wave equations are known (see, for example, Engelbrecht, 1983).
The celebrated KdV equation arises quite naturally for nonlinear dispersive processes. Its standard form reads

$$u_t - uu_x + d_3 u_{xxx} = 0, \tag{3.26}$$

where x and t are the moving frame coordinates and d_3 is the dispersion parameter. In case of nonlinear microstructured solids describe by Eqs. (3.14) – (3.15) the evolution equation in scaled variables yields (Randrüüt, Braun, 2010)

$$u_t + \frac{1}{2}k(u_x^2)_x + (1 - \gamma_1^2)u_{xxx} + \frac{1}{2}m(u_x^2)_{xx} = 0, \tag{3.27}$$

where k, m are the nonlinear parameters for macro- and microstructure, respectively;
$\gamma_1 = c_1/c$ is the ratio of velocities in macro- and microstructure.
In dilatant granular materials the governing evolution equation has a hierarchical structure (Giovine, Oliveri, 1995; Ilison, Salupere, 2009):

$$u_t + uu_x + d_3 u_{xxx} + \beta(u_t + uu_x + d_5 u_{xxx})_{xx} = 0. \tag{3.28}$$

Here two KdV operators are involved, both with their dispersion parameters d_3 and d_5, while βincludes the ratio of the grain size and the wavelength.
In martensitic – austenitic alloys the process is governed by more complicated nonlinearity and higher-order dispersive terms (Salupere et al., 2001; Ilison and Salupere, 2006)

$$u_t + (-u + u^3)u_x + d_3 u_{xxx} + d_5 u_{5x} = 0. \tag{3.29}$$

This equation has quadratic-quartic nonlinearity and third- and fifth-order dispersion terms (cf. Eq. (3.28)).
In presence of external forces, the KdV equation can be presented like (Engelbrecht, 1991; Engelbrecht, Salupere, 1991):

$$u_t + uu_x + d_3 u_{xxx} = f(u), \tag{3.30}$$

where $f(u)$ is a smooth function.

In case of twodimensional wave beams, the evolution equation for $u(x, y)$ is of the Kadomtsev-Petviashvili type:

$$\frac{\partial}{\partial x} R(u) = n u_{yy}, \tag{3.31}$$

where $R(u)$ is a one-dimensional operator and n is a coefficient which characterizes the transverse diffractional effects. In case of the microstructured solid (see Eq. (3.27)), the final form is (Sertakov et al., 2014)

$$\left[u_t + \frac{1}{2} k (u_x^2)_x - (1 - \gamma^2) u_{xxx} + \frac{1}{2} m (u_x^2)_{xx} \right] = n u_{yy}. \tag{3.32}$$

Finally, it is possible also to derive an evolution equation for a nerve pulse (Engelbrecht, 1981). In proper variables we obtain then

$$u_{\xi x} + f(u) u_\xi + g(u) = 0, \tag{3.33}$$

where $f(u)$ is a quadratic function and $g(u)$ is a linear function. The governing equation for a steady wave $u(x + c\xi)$ is then given by (Engelbrecht, 1991)

$$u'' + f(u) u' + c^{-1} g(u) = 0, \tag{3.34}$$

which is of the Liénard type.

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Numerics

There are but a few analytical solutions to complicated wave equations under arbitrary initial and/or boundary conditions. That is why numerical methods must be applied for the analysis of wave propagation. The attention is certainly paid to the accuracy and convergence (if needed) of numerical simulation in order to guarantee the accuracy of description of physical effects. Two methods: (i) the finite volume method and (ii) the pseudospectral method are central although in some cases also other methods are used like the perturbation technique, the Laplace transform, etc.

(i) the finite volume method. Actually under this approach a combination of continuum mechanics, the thermodynamics of discrete systems and finite-volume numerical methods is developed. Using the similarity between the discrete representation of conservation laws and the thermodynamics of discrete systems, the Godunov-type numerical schemes for simulation of wave and front propagation are reformulated (Berezovski, Maugin, 2001;2002). The core idea is to use excess quantities which appear in the local equilibrium approximation due to the interaction between discrete systems. As a consequence, a thermodynamically consistent numerical algorithm is developed (Berezovski et al., 2008).

(ii) the pseudospectral method. Based on the discrete Fourier transform, the pseudospectral method (Orszag,1972) is used for integrating the wave equations and evolution equations. It is not only the accuracy of the method and low computational costs that make this method useful in the analysis of wave processes but also the calculated Fourier transform related spectral characteristics which carry additional information about the internal structure of waves. The practical applications need a careful analysis of filtering and the modificartion of the method for dealing with mixed derivatives in wave equations (Salupere, 2009).
Finally, the practicalities on several numerical methods are collected by Tamm et al. (2015).

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Physical effects and highlights

Modelling. The novel mathematical models are derived and analysed:

- Dual internal variables are introduced in order to describe the microstructure(s) in solids; this approach allows modelling the multiscale structure of such solids and accompanying temperature effects including the simultaneous effects of the elastic and thermal properties of the microstructure; the importance to account the inertia of a microstructure is demonstrated.
- The stress-strain relations for fibre reinforced materials and wool felt are derived.
- The description of spins and twist waves in mesoscopic continuum mechanics is proposed.
- Wave hierarchies are described depending on scale parameters.
- The conditions for the existence of normal and anomalous dispersion are determined.
- Negative group velocity (NGV) is established for hierarchical microstructures with a conjecture that this phenomenon is caused by a pre-resonant situation of optical modes.
- Inverse problems of non-destructive evaluation (NDE) of material properties are solved by using special elaborated algorithms.
- A novel evolution equation describing the nerve pulse propagation is derived.
- An improved model describing the longitudinal waves in biomembranes is proposed.

Solitonic structures. Using the two-wave and one-wave (evolution) equations, novel physical effects are established:

- The long-time behaviour of the KdV soliton interaction has revealed the rhombus-type pattern of trajectories and the sequence of unperturbed emerging solitons follows the Farey tree distribution.
- The existence of hidden solitons is demonstrated for the KdV equation and mechanisms of emergence of hidden solitons under external forces are established.
- The solitonic structures for modified KdV- type equations (KdV 435) and the hierarchical KdV equation are found including plaited solitons, soliton ensembles, solitons accompanyed by wave packets, etc.
- The principal differences on wave profiles of solitary waves between the deformation-dependent (elastic solids) and displacement-dependent (biomembranes) nonlinearities are analysed.
- The conditions of the existence solitonic solutions in two-wave Boussinesq-type equations are established.
- The emergence of solitonic trains in two-wave systems is demonstrated.
- Interaction mechanisms for solitonic structures are established.
- Novel 1D and 2D evolution equations for waves in microstructured solids derived.
- The novel idea to use deformed solitons for the NDE of microstructured solids is proposed.

Numerics. The derived finite volume method is thermodynamically consistent and therefore permits to solve several problems concerning thermal effects while the pseudospectral method permits to calculate spectral characteristics casting additional information on physical effects. Among the solved problems are:

- Phase transformation in austenitic-martensitic alloys.
- Wave propagation in FGMs and layered materials including the emergence of solitary waves.
- The scattering of elastic waves on elastic scatterers and establishing the possible energy concentration in such processes.
- Establishing wave-like behaviour of thermal processes in microstructured materials.
- Analysing the propagation of wave fronts and Mode I-type cracks.
- Interaction of piano hammers and vibrating strings are analysed by using novel mathematical models.

Chapter 4
Fields in solids

Preliminaries

In manufacturing very often the prestress fields remain in solids which may affect the future loading of structural elements. For determining these prestress fields, several non-destructive methods are then used. In CENS, the methods of *photoelasticity* have been developed for a long time (see Chapter 2) and were successfully continued. In terms of fields, photoelasticity is based on interaction of stress fields with light field. In addition, *acoustodiagnostics* is based on interaction of stress fields with ultrasonic waves, i.e. with acoustical waves. Both methods are successfully used for non-destructive testing (NDT). Finally, the interaction of deformation and thermal field in microstructured solids is modelled within the framework of *thermoelasticity*.

Photoelasticity

The basic equations are the following. In integrated photoelasticity polarised light is passed through the 3D anisotropic and inhomogeneous test object and transformation of the polarisation is recorded. It has been shown that the following equations describe adequately the transformation of the polarisation (Aben, 1979)

$$\frac{\mathrm{d}E_1^P}{\mathrm{dz}} = -\frac{1}{2} i C_0 (\sigma_1 - \sigma_2) E_1^P + \frac{\mathrm{d}\varphi}{\mathrm{dz}} E_2^P, \tag{4.1}$$

$$\frac{\mathrm{d}E_2^P}{\mathrm{dz}} = -\frac{\mathrm{d}\varphi}{\mathrm{dz}} E_1^P + \frac{1}{2} i C_0 (\sigma_1 - \sigma_2) E_2^P, \tag{4.2}$$

Here E_1^P and E_2^P are components of the electric vector along the principal stress axes in the plane x_1, x_2, σ_1 and σ_2 are the principal stresses in this plane, and dφ /dz is rotation of the principal stress axes along the light ray.,

If principal stress directions on the light ray are constant, in Eqs. (3), (4) we have dφ /dz = 0. Now the integral Wertheim law is valid:

$$\Delta_* = C_0 \int (\sigma_1 - \sigma_2) \mathrm{dz}. \tag{4.3}$$

From Eq. (4.3) it follows, that if stresses along the light ray are in equilibrium, the integral is zero and no photoelastic effect can be observed. That excludes direct measurement of stress in bent plates and shells and also in tempered glass panels.

In magnetophotoelasticity, if the test object is placed in a magnetic field, which is parallel to the light propagation direction, the Eqs. (3), (4) take the form

$$\frac{\mathrm{d}E_1^P}{\mathrm{dz}} = -\frac{1}{2} i C_0 (\sigma_1 - \sigma_2) E_1^P + \frac{\mathrm{d}(\varphi - \psi)}{\mathrm{dz}} E_2^P, \tag{4.4}$$

$$\frac{\mathrm{d}E_2^P}{\mathrm{dz}} = -\frac{\mathrm{d}(\varphi - \psi)}{\mathrm{dz}} E_1^P + \frac{1}{2} i C_0 (\sigma_1 - \sigma_2) E_2^P, \tag{4.5}$$

where ψ is rotation of the plane of polarisation due to the Faraday effect. Now, even when dφ/dz = 0, the integral Wertheim law (4.3) is not valid any more, the characteristic parameters α_0, α_* and Δ_* can be measured and certain information about the stress distribution, which is in equilibrium, can be obtained. Magnetophotoelasticity is a sophisticated method of 3D stress

measurement, used in specific cases (Aben, 1970; Ainola, Aben 2004). E. g., Pilkington glass company has used it for measuring residual stress in windshields (Clarke et al., 1999).
Below several resent theoretical results and applications are briefly described.

Integrated photoelasticity as hybrid mechanics. In experimental mechanics often the measurements do directly not give all the needed information about the stress field. In many cases elaboration of the measurement data using a mathematical model may give complete solution of the problem. This is named hybrid mechanics. A hybrid mechanics method has been elaborated for the determination of axisymmetric residual stress fields in glass since integrated photoelasticity measurements alone are not sufficient. As an additional equation, the classical sum rule, derived by O'Rourke(1951), has been generalized for the case when stress gradient in the axial direction is present. The **generalized sum rule** opens up the possibility to determine completely axisymmetric residual stress fields in glass (Ainola, Aben, 2000).

Photoelastic tomography. While for determining the axisymmetric stress field it is needed to pass polarized light through the test object only parallel to one direction, to measure 3D stress fields in the general case integrated photoelastic measurements are to be carried out in many directions. Thus the classical tomography, which deals with scalar fields, is to be generalized for stress tensor fields. The complicated problem of tensor field tomography was reduced to a problem of scalar field tomography for a single component of the stress tensor (Ainola, Aben, 2000). By that the linear approximation of the equations of integrated photoelasticity was used (Aben, Guillemet, 1993). Later on a nonlinear algorithm of photoelastic tomography, based on nonlinear equations, was developed [Aben, Errapart, 2007, 2012). In the latter an algorithm of genetic programming, the method of differential evolution, was used. Tomographic measurement of the laser's Gaussian electric field [(Aben, Ainola, 2001) and of the Kerr effect have also been studied. The generalization of the Abel inversion for tensor fields is considered (Aben et al., 2010)

Photoelasticity of glass. The aim of the research of the laboratory of Photoelasticity has been both to develop the theory and methodology of 3D photoelasticity and to give to glass industry new and efficient methods for quality control (Aben et al., 2002; 2003). For the latter aim, in 2003 a spin-off company Glasstress Ltd was founded for the manufacturing and marketing of the apparatus for glass stress measurement. Until today more than 30 integrated photoelasticity polariscopes AP and more than 70 polariscopes SCALP for the measurement of stress in architectural glass panels and automotive glazing have been sold to different glass companies and universities all over the world. The polariscope SCALP is based on the scattered light method. A review of the state of the art of glass stress measurement is given in (Aben et al., 2008; 2014).

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Nonlinear acoustodiagnostics.

Advanced ultrasonic nondestructive material characterization methods based on new principles are developed in this project, with special interests on dynamic processes of wave propagation and interaction in materials with smoothly changing properties. Novelty includes a wide usage of nonlinear effects of wave-wave, wave-material and wave-prestress interaction that enables to elaborate more informative methods in comparison with the conventional methods for nondestructive characterization of materials with complex properties.

The problem of wave propagation in prestressed material is treated as a quasi one-dimensional: one-dimensional longitudinal wave is propagating in the material subjected to two-dimensional prestressed state (Ravasoo, Lundberg, 2001). The wave and the prestress are characterized by displacement $u_1(x_1,t)$ and $u^{\mathbf{0}}_{i,j}(x_1,x_2)$, respectively. The wave propagation is governed by the nonlinear equation of motion (Ravasoo, 2007)

$$[1 + k_1 u^0_{1,1} + k_2 u^0_{2,2}]u_{1,11} + [k_1 u^0_{1,11} + k_3 u^0_{1,22} + k_4 u^0_{2,12}]u_{1,1} + \\ + k_1 u_{1,11} u_{1,1} - c^{-2} u_{1,tt} = 0, \qquad (4.6)$$

where indices 1, 2 and t after a comma indicate differentiation with respect to Lagrangian rectangular coordinates x_1 and x_2, and time t, respectively and k_i , i=1,2,3,4 are material parameters. The two-dimensional prestressed state is described by a set of two static balance laws. Equation (4.6) is a typical governing equation describing the interaction of the prestress field with a propagating wave.

The algorithms for qualitative and quantitative nondestructive characterization of two-parametric prestress in the physically nonlinear elastic materials (structured elements) are elaborated making use of the fact that the phase velocities and amplitudes of harmonics of the propagating ultrasonic harmonic wave are sensitive to the prestress (Ravasoo, 2009; Ravasoo, Braunbrück, 2007). It was clarified that the maximum information about the prestressed state may be extracted from the wave propagation data by choosing the values of two essential small parameters in the problem to be of the same order (Ravasoo 2010).

Different algorithms for qualitative ultrasonic nondestructive characterization of FGMs with strongly and smoothly changing physical properties are waves (Ravasoo,2011) or bursts

(Ravasoo, 2012) is excited on the boundaries of the specimens of different FGMs. The variation of material properties is reverberated in boundary oscillation profiles. The influence of the variation of material properties on the modulation of boundary oscillations is studied and analysed comparing the computed wave profiles with the corresponding profiles on the boundaries of the homogeneous isotropic material (Ravasoo, 2011) [10-12] or resorting to the phase plots and their generalization into parametric plots [5,6].
Two novel ideas for the NDT must be stressed: the usage of counterpropagating waves (ie two detecting wave fields) and the informative phase plots or parametric plots for the analysis.

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Thermoelasticity
The classical theory of thermoelasticity describing the coupling of wave motion (deformation field) and thermal field is well known. Mathematically it means the coupling of a hyperbolic and a parabolic equations. The situation in microstructured solids is even more complicated because the microstructure compared with the macrostructure has, as a rule not only different elastic characteristics but also different thermal properties. This means that one has to account for microdeformation and microtemperature (fluctuations of temperature due to the microstructure) simultaneously. Based on the theory of dual internal variables which model the internal fields, it is possible to derive the governing equations for waves in such microstructured solids (Berezovski et al., 2014; Engelbrecht, 2015):
balance of (linear) momentum

$$\rho_0 u_{tt} = (\lambda + 2\mu) u_{xx} + m\theta_x + A\alpha_x + M\varphi_{xx}, \tag{4.7}$$

balance of energy

$$\rho_0 c_p \theta_t = (k\theta_x)_x + m\theta_0 u_{xt} + \frac{R_{22}}{R_{12}^2}\varphi_t^2, \tag{4.8}$$

governing equation for microdeformation

$$I\alpha_{tt} = C\alpha_{xx} - Au_x - B\alpha, \tag{4.9}$$

governing equation for microtemperature

$$I_t \varphi_{tt} + \frac{R_{22}}{R_{12}^2} \varphi_t = N\varphi_{xx} + Mu_{xx} \,. \tag{4.10}$$

Here u is the macrodisplacement, α is the microdeformation, θ is the macrotemperature and φ is the microtemperature. The material constants are: ρ_0 is the density, λ, μ are Lamé parameters, θ_0 is the initial temperature, c_p is the specific heat, I is the inertia of the microstructure and I_t is an internal inertia measure. The other constants $A, B, C, M, N, R_{22}, R_{12}, m$ reflect the material parameters (Berezovski et al., 2014).
The model (4.7)-(4.10) contains beside the wave equation (4.7) two hyperbolic equations: Eq (4.9) for the microdeformation and Eq. (4.10) for the microtemperature. These equations are not directly coupled, but both of them are coupled with the balance equation (4.7). The heat conduction equation (4.8) is only affected by the microtemperature field. The coupling can induce wave-like propagation of macrotemperature.This is demonstrated by numerical calculations for the simpler case when microtemperature is negleted (Berezovski, Engelbrecht, 2013).

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Chapter 5
Water waves and interactions

Preliminaries

The studies in wave engineering including waves and vortices on sea, started earlier in 90ies in the Institute of Marine Systems (see Chapter 2) and got new impetus after joining CENS in 1999 as a working group and later when T. Soomere joined the Institute of Cybernetics in 2005. The Laboratory of Wave Engineering was launched in 2008. The range of studies has been widened – from water waves the attention was also turned to coastal protection, marine fairways, wave climatology, pollution and patches, etc. It means that the analysis accompanied by experiments demonstrates the full picture of surface waves in general and particularly as wind-or ship-driven processes, the wave climatology and its changes in space and time, and the impact of waves on coastal areas.

Rossby waves and kinetic equations

Analysis of the results of an experimental study of Rossby wave interactions in the "Coriolis" rotating tank (diameter 13m, Grenoble University) confirmed that travelling waves excited a noticeable zonal transport in the form of weak jets after ca 10 wave periods. The experiments confirm the prediction of the weakly nonlinear (kinetic) theory that propagation of Rossby waves may result in a considerable zonal transport of water masses (Soomere, Koppel, 1999). In several experiments with anticyclones, an unexpected interaction of vortex dipoles has been observed. Namely, concentrated anticyclons penetrated into cyclons and formed practically axisymmetric mostly cyclonic structures with a strong anticyclonic kernel and an intermediate zone with nearly zero vorticity. In several experiments concentrated anticyclones demonstrated a bizarre tendency to penetrate into relatively large cyclones. Detailed study of the above-mentioned peculiarity of the synoptic motions has revealed the existence of generalised anisotropic, thermodynamically equilibrated spectra of such motions. Owing to the "balance" of the nonlinearity. Rossby-wave systems evolve towards a particular equilibrated state, consisting of a superposition of a zonal flow and a spectrally isotropic wave system (Soomere, 1999).

The equilibrium solutions to kinetic equations of weak turbulence (weakly nonlinear wave systems) are analysed in a systematic manner The most important consequence is that systems with four-wave resonance conserve the particle number (wave action) whereas systems with three-wave or five-wave resonance violate this law (Soomere, 2001).It is shown that in the framework of multi-modal kinetic equations (describing resonant energy exchange between different wave classes) energy exhange rate depends ontwo different types of coefficients - interaction coefficients and coupling coefficients. As an example of multi-modal kinetic equations describing evolution of different wave systems with comparable frequencies, the kinetic equation describing slow evolution of the energy spectrum of baroclinic Rossby waves in three-layer model ocean is derived. Explicit analytical expressions for the coupling coefficients describing energy exchange intensity between different modes are obtained and their main properties are established (Soomere, 2003).

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Surface waves

The inverse problem for wave crests is introduced. A solution strategy for two-waves interactions is given. Taking the model of water waves the KP (Kadomtsev-Petviashvili) equation, that describes (small but finite amplitude, long) surface waves travelling in shallow water (of constant depth) „mainly" in one direction, it is proved that the inverse problem has a unique solution. Actual solutions for the KP two-wave interactions are constructed, in particular for the cases with small interaction angle, moderate phase shifts, and/or symmetric interactions. Sensitivity of the inverse wave crest problem is investigated. As a result, the question of the practical applicability of the method is answered: the method of calculating the heights of the waves from a "photographic" record is applicable if the positions of the waves crests can be well defined from the wave pattern and the interaction angle is not too small (Peterson, van Groesen, 2000; 2001). As a result of this study multi-soliton solutions have been constructed for KdV type equations (in Hirota sense) using the Hirota bilinear formalism. A novel multi-soliton decomposition is proposed and analyzed. This decomposition is useful for describing the interactions between many solitons and it supports the emergence of interaction soliton(s). With the results of this study these complex interactions are completely described and can be used to predict the evolution of non-stationary multi-soliton interaction patterns. For that an algorithmic method for constructing multi-soliton interaction pictures is derived (Peterson, 2002). For the single interaction process, the wave system is decomposed into the incoming and the interaction soliton that represents the particularly high wave hump analogous to Mach stem in the crossing area of the waves. Shown is that extreme surface elevations up to four times exceeding the amplitude of the incoming waves typically cover a very small area but in the near-resonance case they may have considerable extension (Peterson et al., 2003). In addition, it is shown how the interaction process looks like with unequal amplitudes (Soomere, 2004). Is is also shown that the slopes of an interaction soliton may exceed eight time the slopes of the incoming soliton (Soomere, Engelbrecht, 2005). In shallow sea areas near-resonant interaction of solitonic surface wave systems with radically different amplitudes apparently becomes evident in the form of bending of crests of the larger waves. This phenomenon may drastically increase the probability of encountering a hit by a high wave possibly with a particularly large slope and arriving from an unexpected direction.

The general overview on soliton intractions and the analysis of emergence of rogue waves are presented (Soomere, 2009; 2010).

An efficient and highly accurate solver for 2D free surface problem is developed using half analytical/half numerical approach based on conformal mapping technique. The solver is successfully tested against the exact solution (Stokes waves) of the free surface problem over a large number of wave periods with a very high accuracy, stability, and efficiency of the numerical algorithm (Peterson).

The solitary waves may occur not only at surface but also like internal waves in multilayered fluids (Kurkina et al., 2015).

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Wake waves

The waves from fast ferries have become a problem of growing concern in the vicinity of ship lanes during the last years. The role of ship waves is impressive in terms of energy (it forms about 10% from the bulk wave energy) but striking in terms of energy flux or wave power: ship waves form 18-35% (27-54% during the summer season) from the total wave power at the coasts of Tallinn Bay. The highest components of ship wakes with the heights of about 1 m have frequently periods 10-15 s that are considerable larger than periods of natural waves (typically 2-5 s, in extreme cases 6-7 s) in this area. (Soomere, Rannat, 2003). Such high and long surface waves do not exist in natural conditions in the area in question. They cause unusually high near-bottom velocities at the depths of 5-30 m, thus forming a new forcing component of vital impact on the local ecosystem that may cause considerable intensification of beach processes as well as enhanced vertical mixing in the water body (Soomere, 2005). The shape and properties of long ship-generated waves approaching shallow coastal areas of Tallinn Bay are studied based on recordings of water surface time series. For typical leading wake waves nonlinear effects become significant at depths of 10 - 15 m. A large part of waves (with the height of >0.4 m) have the shape of cnoidal waves in shallow areas with depths of 4 - 5 m. The shapes of the largest wake waves are close to the solitary wave solutions of the Korteweg de Vries equation. Such waves excite considerably larger velocities of water particles than sinusoidal waves of the equal height and length. (Soomere et al., 2005).

The periods of wake waves from high-speed ships frequently are much larger than dominating periods of wind waves. The leading waves typically have a height of about 1 m and a period of 10-15 s. Such waves extremely seldom occur in natural conditions in many regions of semi-enclosed seas. They cause unusually high hydrodynamic loads in the deeper part of the nearshore. The fast ferry traffic thus is a qualitatively new forcing component of vital impact on the local ecosystem. Wakes from high-speed ferries may trigger considerable changes of the existing balance of coastal processes (Soomere 2006).

Variability in the properties of wakes generated by high-speed ferries is analysed. The data from > 400 wakes are used for the construction of empirical probability distribution functions of different wake properties (maximum height, wake energy, and energy flux). The periods of the highest waves vary insignificantly and are closely related to the cruise speed of the vessels. An appropriate measure of the properties and variability of wakes is the maximum wave height. Wakes from ’classic’ high-speed ships are very variable. Wakes from large, basically conventional, but strongly powered ferries show quite limited variability, thus, both the average and extreme wake properties of such ships can be more easily adjusted by changing their sailing regime (Parnell et al., 2008: Torsvik, Soomere, 2008; Kurennoy et al., 2009).
A substantial part of the energy of wake waves from high-speed ships is concentrated in nonlinear components which at times have a solitonic nature. Recent results of investigations into solitonic wave interactions within the framework of the KP equation and their implications for the rogue wave theory are reviewed in a systematic manner The optional nonlinear components of the ship wakes such as the very narrow V-like wake components, packets of monochromatic waves, ship-generated depression areas and supercritical bores are also discussed (Soomere,2007; 2009; Kurennoy et al., 2011).
A new mechanism producing onshore transport of substantial amounts of water remote from the fairway through wake waves generated by high-speed ferries is described based on high-resolution water surface profiling (Torsvik, Soomere, 2009; Torsvik et al., 2009. A novel method making use of spectrogram analysis has been applied to quantify the duration, intensity and frequency distribution of wake waves from high-speed ferries and conventional ships (Didenkulova et al, 2011; Torsvik et al., 2015). The spectogram representation offers a convenient way to identify a specific signature of single types of ships. The method is described in detail by Torsvik, Soomere et al. (2015).
The mechanisms of generation of ship-induced depression waves in the Venice Lagoon are proposed (Parnell et al., 2015).

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Wave climatology

Waves on sea are strongly influenced by winds which may undergo temporal changes. The analysis of wind-driven waves has a direct impact on coastal protection and pollution. That is why the studies of wave climatology have been in focus for a long time.

It is shown that directional distribution of moderate and strong winds in the Baltic Sea area is strongly anisotropic. The dominating wind direction is southwest and a secondary peak corresponds to north winds. Northwest storms are relatively infrequent and north-east storms are extremely rare. The primary properties of the anisotropy such as prevailing winds, frequency of their occurrence, directional distribution of mean and maximum wind speed coincide on both sides of the Baltic Proper (Soomere, Keevallik, 2001). Properties of saturated wave field in the neighbourhood of possible sites of the Saaremaa deep harbour during typical (~15 m/s) and extreme storms (~25 m/s) are analysed on the basis of wave model WAM forced by steady winds, and directional distribution of winds. The highest waves correspond to NNW storms. Remarkable waveheight anomalies may occur in the neighbourhood of the harbour sites. The anomalies emerge only during very strong storms and may serve as a major navigation danger (Soomere, 2001). The wind wave regime of Tallinn Bay, Gulf of Finland, is analysed with the use of a simplified method of long-term computations of wave fields based on a high-resolution nested WAM model, Kalbådagrund (1991-2000) wind data and on a specific technique of splitting of long-term wave calculations into short independent slices (Soomere, 2005).

Actually the studies of wave climatology covered a large eastern coast of Baltic Proper, covering the coast from St Petersburg (Russia) to Klaipeda (Lithuania) demonstrating the trends and regimes of wave regimes (Broman et al., 2006; Keevallik, Soomere, 2008; Kelpšaite et al. 2008; Soomere et al., 2008). A special attention is devoted to the analysis of extreme wave conditions in the Baltic Proper and in the Gulf of Finland during windstorm Gudrun in January 2005. During this storm the significant wave height off the coast of Saaremaa and Latvia was about 9.5 m (Soomere et al., 2008). The shifts in early spring wind regimes in North-East Europe (1955-2007) were established (Keevallik, Soomere, 2008). An important part of the Baltic Sea for Estonia and Finland is the Gulf of Finland. The progress in knowledge of the physical oceanography of the Gulf of Finland is presented for 1997-2007 (Soomere et al., 2008). Another study on extremes and decadal variations is based on long-term time series from Almagrundet (1978–2003) and Vilsandi (1954–2005), and wave statistics from the middle of the northern Baltic Proper which demonstrated average wave conditions, their seasonal cycle and decadal variations, and extreme wave storms in the northern Baltic Sea (Soomere, 2008).

Seasonal and long-term variations of wave conditions in the northern Baltic Sea and the related uncertainties have been analysed by means of merging historical visual observations and numerical hindcasts to reveal the basic features of the wave properties. Wave conditions, their seasonal cycle, and inter-annual and long-term variations are quantified based on (i) visual observations along its eastern coast at Vilsandi and Pakri, (ii) instrumentally measured wave properties at Almagrundet on the western coast, (iii) directional wave statistics from the northern Baltic Proper, and (iv) wave hindcast using a fetch-based point model and a shorter hindcast with the WAM wave model forced by geostrophic and MESAN winds (Zaitseva-Pärnaste,et al., 2009).

The wave climate and its variability in the north-eastern Baltic Sea are estimated using the wave model WAM driven by adjusted geostrophic winds for 1970-2007 under ice-free conditions and a spatial resolution of 3 nautical miles. The hindcast qualitatively reproduces the time series of the sea state and replicates the seasonal patterns of wave intensity and different statistical properties of wave fields in both offshore and coastal regions. It is shown that the hindcast generally underestimates the wave heights even if the very best wind information is used (Räämet, T.Soomere, 2010).

The basic features of the wave climate in the South-Western Baltic Sea are established based on the second longest instrumentally recorded wave time series in the Baltic Sea at the Darss Sill in 1991-2010 (Soomere, Räämet, 2011; Soomere et al., 2012) and also for the Eastern part of the Baltic Sea (Zaitseva-Pärnaste et al., 2011). The wave energy resource theoretically and practically available in a semi-sheltered shelf sea of moderate depth and with highly intermittent wave climate has been quantified on the example of the Baltic Sea. Wind wave climatology in the eastern part of the Baltic Sea has been extended back to 1946 (Soomere, 2013) and the first approximation of a similar climatology for Lake Peipsi has been constructed.

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Coastal protection, run-up waves and sediments

The run-up of waves on beaches affects considerably the sediment transport, that is why the run-up mechanisms must be carefully analysed (Didenkulova et al., 2007) which is even more important for tsunami waves (Didenkulova, 2008). The basic factors affecting sediment supply for and transport processes at Pirita Beach (Tallinn) and the Narva beach at the mouth of Narva River are analysed (Soomere et al., 2007).

Shoaling and run-up of long waves induced by high-speed ferries in Tallinn Bay is examined theoretically and experimentally, focusing on the dependence of runup height on the incident wave properties. Experimental data from 212 wake events in Tallinn Bay demonstrate that the largest ship generated waves approaching the coast break in the nearshore and have only weak wave amplification at the beach (Didenkulova et al., 2009).

Vessel-wave induced potential has a strong influence on long shore sediment transport. The wind-wave time series at the SW coast of the Island of Aegna in 1981–2008 is modelled on the basis of a simplified scheme for a long-term wave hindcast with the use of a triple-nested version of the WAM model. Longshore drift created by waves of different origin is estimated by the CERC energy flux model. Vessel wakes cause longshore drift that had in 2007–2008 a magnitude about 25% of and opposite directed to that produced by wind waves (Kelpšaite, et al., 2009).

The qualitative patterns of wave-driven net and bulk sediment transport along the eastern Baltic Sea coast are very robust. The overall counter-clockwise transport contains two persistent reversals (Viška, Soomere, 2013; Soomere, Viška, 2014). It is shown that beach profiles may develop a two-section almost-equilibrium structure under joint impact of short wind waves and groups of long ship waves. The upper section of a profile is convex and follows the 4/3 power law at small depths and in the swash zone (Didenkulova, Pelinovsky,

2012). The maximum wave set-up (up to 70-80 cm) forms more than 50% of all-time maximum water level and thus may serve as a substantial source of marine hazard for low-lying regions within and around Tallinn (Didenkulova et al., 2011).
The options for using an ensemble of projections to evaluate return periods of extreme water levels are established for selected locations of the Estonian coast. Statistical parameters of the wave inundation on a plane beach are calculated within nonlinear shallow water theory and studied experimentally. The probability of coastal floods grows with an increase in the nonlinearity of the incident wave field. It is important to be aware of mitigation risks (Soomere, 2013).

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Marine fairways and pollution

Sea coasts located adjacent to fairways hosting intense ship traffic are frequently subject to major oil pollutions. The possibilities of the choice of the fairway (or smart re-routing ofthe ship traffic) so that a potential oil spill will stay in the open sea area as long as possible are analysed based on certain intrinsic properties of dynamics of water masses – specific patterns of subsurface currents in the Gulf of Finland. Perspectives and consequences connected with solutions of this type are discussed (Soomere, Quak, 2007).
Extensive statistical analysis of trajectories of current-driven surface transport in the Gulf of Finland, the Baltic Sea, is carried out for the period of 1987-1991, with the goal to construct a map of probabilities for adverse impacts released in different sea areas to hit the coast and to establish the offshore areas that are statistically safe to travel in. The properties of the net transport of surface water and time scales for reaching the nearshore of pollution released in different areas of the Gulf of Finland, the Baltic Sea, are analysed based on Lagrangian

trajectories of water particles reconstructed using the TRACMASS model from three-dimensional velocity fields calculated by the Rossby Centre (Swedish Hydrological and Meteorological Institute) circulation model for 1987–1991 (Soomere et al., 2010).
A novel method is proposed for the optimization of marine fairways, based on the quantification of various offshore areas according to the probability of pollution released in these areas to reach vulnerable regions and tested for the Gulf of Finland. (Delpeche-Ellmann, Soomere, 2013; Giudici, Soomere, 2013).This technique is expanded to cover also South-Western Baltic Sea and Kattegat (Lu et al., 2012). The analysis of the potential pollution released during a ship accident and further carried by currents may affect marine protected areas in the Gulf of Finland at very large distances up to 200 km. Shifting the major fairway by a small distance in some sections may lead to a huge decrease in the amount of pollution carried to the largest marine projected area (Soomere, Quak, 2013; Viikmäe, Soomere, 2014). The mechanisms of the formation pollution patches are established (Giudici, Soomere, 2014; Kalda et al., 2014). The research into pollution mechanisms opens the ways to better environmental analysis (Soomere et al., 2014).

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International waters

Although the main attention in wave climatology has been to the Baltic Proper and its Eastern part including Estonian, Latvian and Lithuanian coast, many studies have been devoted to international waters. Some examples follow.
Extreme waves generated by cyclones in Guadaloupe are analysed and the most dangerous regions evaluated (Zahibo et al., 2008).
The internal waves in the Saint John River Estuary, New Brunswick, Canada have been analysed and mechanism of the diapycnal mixing established (N.Delpeche et al., 2010).

The ideas and calculations estimating an optimal marine fairway in the Gulf of Finland are used also for the South-Western Baltic and Kattegat (Lu et al., 2012)
The mechanisms of generation of depression waves in Venice Lagoon are described and the properties of such depressions established (Parnell et al., 2015).

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Chapter 6
Systems biology

Preliminaries
The studies in the extremely perspective field of systems biology has started already in the 90ies (see Chapter 2) and got a strong impact by establishing the Laboratory of the Systems Biology in 2008 and by the support of the Wellcome Trust grant " Analysis of structural and functional aspects of compartmentation of adenine nucleotides in heart muscle cells" (2007-2012 with the prolongation up to 2014). The research was focused on cardiac energetic from the viewpoint of the regulation of intracellular processes and understanding the intracellular interactions. Studies were based on the strong symbiosis between the experimental analysis, biophysical (mathematical) modelling and computational (*in silico*) analysis. In general terms, the mechanisms of intracellular interactions reflect clearly the complexity of physiological processes – biocomplexity.

Cardiac energetics
(i) cellular energetics and energy fluxes. Quantitative (mathematical) methods were derived of compartmentalised energy fluxes of living cells. The aim was to understand which of the intracellular metabolites may regulate the oxidative phosphorylation (OxPhos) under various enzymic conditions of the cell. The role of creatine kinase and adenylate kinase was established (Kongas et al, 1999). For chemically skinned cardiac cells it has been demonstrated that the affinity of fibres may depend on the ratio of the velocity of synthesis of ATP in mitochondria to the activity of nonspecific background ATPases (Kongas et al., 2002). The regulation of mitochondrial respiration in heart cells was analysed by the reaction-diffusion model for energy transfer (Vendelin et al., 2000) and the role of the creatine-phosphocreatine system explained (Saks et al, 2000). A hypothesis is proposed that the low affinity of the fibres to ADP is caused by the restricted diffusion into the fibre. Experiments show the validity of this hypothesis and in addition, the ideas how to estimate diffusion barriers between the ATP producing and consuming sites in cardiac cells are proposed (Kongas, et al. 2004).
The mechanisms which could reproduce the available experimental data on functional coupling between mitochondrial creatine kinase (MiCK) and adenine nucleotide translocase (ANT) are analysed. A model which assumes the direct transfer of substrates between MiCK and ANT are in good agreement with experiments reproducing the measured constants and the estimated ADP flux (M.Vendelin et al., 2004). The arrangement of mitochondria in heart muscles is studied. The results show that intermyofibrillar mitochondria are arranged in a highly ordered crystal-like pattern in a muscle specific manner wwith relatively small deviation in the distances between neighbouring mitochondria. This is consistent with the concept of the unitary nature of the organisation of the muscle energy metabolism (Vendelin et al., 2005). There is a certain difference, however, between mitochondrial positioning in rats and trouts estimated by experiments (Birkedal et al., 2006). The analysis of the ADP compartmentation reveals coupling between pyruvate kinase and ATPases in heart muscle (Sepp et al., 2010).
The analysis of local recovery of sarcoplasmic reticulum (SR) calcium release suggests that local refilling of SR controls calcium spark amplitude recovery (Ramay et al, 2011). It is shown that the dynamic method provides a measure of total flux and not the net flux as presumed previously. It is demonstrated that ADP potently restores calcium retention capacity in severely stressed mitochondria (Sokolova et al., 2013). This effect is most likely not related

to a reduction in reactive oxygen species production. It is shown that in oxidative muscle such as a heart, some ATPases are tightly coupled to glycolosis and do not use ATP provided by mitochondria (Birkedal et al., 2014).

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(ii) diffusion. The intracellular diffusion restrictions have an important role in bioenergetics and could shape the energy transfer in the heart. The existence of diffusion restrictions in trout cardiomyocytes is experimentally shown. The lack of creatine effect indicates that the trout heart lack mitochondrial creatine kinase tightly coupled to respiration whic argues against diffusion restriction by the outer mitochondrial membrane (Sokolova et al, 2009).

It is demonstrated that intracellular structures impose significant diffusion obstacles in rat myocytes using a single cell preparation (Jepihhina et al., 2011). Results obtained in experiments indicate that diffusion of a smaller molecule is restricted more than that of a larger one, when comparing diffusion in cardiomyocytes to that in the solution (Illaste et al., 2012) The presence of periodic intracellular barriers has been suggested. It is demonstrated that the healthy heart of creatine-deficient mice is able to preserve cardiac function at a basal level in the absence of creatine kinase-faciliated energy transfer without compromising intracellular organization and the regulation of mitochondrial energy homoeostasis. It is suggested that at least a part of the diffusion restriction at the mitochondrial outer membrane level is not by the membrane itself but due to the close physical association between the sarcoplamicc reticulum and mitochondria (Sepp et al., 2014).

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(iii) contraction. A model is derived for estimating the local oxygen consumption of the cardiac muscle from the deformation measurements. The Huxley-type model is able to reproduce the amount of cross-bridges that detach without hydrolyzing ATP molecule and estimate correctly the ATP consumption rate (Vendelin et al., 1999). A finite-element model for computing deformation, local strains and stresses and the ATP consumption of left ventricle is developed (Vendelin et al., 2000). It is shown that the physiological fibre orientation in the ventricular wall is optimal for the high ejection rate and the relatively homogeneous distributions of the strain, stresses and the ATP consumption (Vendelin et al., 2002).
An efficient and accurate method for estimating a mean sarcomere length of a single contracting cardiomyocyte is developed on the basis of experiments and numerical calculations (Peterson et al, 2013). This opens a way to understand better the contraction process of myocardium. A cross-bridge model describing the mechanoenergetics of actomyosin interaction is built up which describes cross-bridge ensembles in thermodynamically consistent way (Kalda et al., 2012; 2015).

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(iv) cardiac signals. Studies were devoted to signals generated by the sine node and to the heart rate variability (the EKG signals).

Using a simple nerve pulse equation (see Chapter 3) for modelling signals transmitted by Purkinje fibres, the harmonic response has revealed general bifurcation regimes which can explain certain arrhythmias (Kongas, et al., 1999; Kongas,2000). The mechanism of co-existing attractors is also explained.

The analysis of the scale-invariant properties of the distribution of the low-variability periods of heart rate variability (HRV) has been performed (Kalda, et al; 2001a, Säkki et al., 2003). It has been concluded that for healthy patients, the distribution typically follows the Zipf law; for other groups of patients (with post myocardical infarction, hypertension, etc) the multiscaling behaviour is typical (Kalda et al., 2001b). The rank-length distribution law of the low-variability periods has been used to construct three new non-linear measures of the HRV (Säkki et al., 2004). These quantities characterize the complex structure of the HRV signal where the low- and high-variability periods are deeply interwined. Analytical approach has been used to compare and classify the most important measures of HRV including inter-beat interval increment distribution (Säkki et al., 2003). For healthy subject, this distribution corresponds to a stationary Lévi distribution. An analytic relationship between the multifractal spectrum of intermittent time series and the multiscaling exponent describing the length-distribution of low-variability periods is derived (Kalda et al., 2004).

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*(v) methods.*The theoretical, experimental and simulation methods are developed within the studies of cellular energetic and systems biology in general.

The general principles of mathematical modelling of complex biological systems are analysed within the framework of cellular energetic (Vendelin et al, 2007).

An open-source package for deconvolution of confocal microscopy images is developed on the basis of Richardson-Lucy algorithm and stopping criteria for deconvolution of images have been established (M.Laasmaa et al., 2011). A symbolic Gauss-Jordan elimination routine for analyzing large metabolic networks has been developed.

Experimental methods have been developed in the special laboratory equipped using funding from the Wellcome Trust and EU initiatives. There are two wide-field fluorescence microscopes with single-cell mechanics and electrophysiology setups, a phonon counting confocal microscope, respiration chambers, and absorbance spectrophotometer. The microscopes have been custom designed and programmed using a software platform developed in the group. This platform provides freedom to control and automate the course of

various experiments and extend the setups. There is an access to additional required equipment within the TUT, as well as an access to a high-performance computer cluster in the TUT.
Special software (F2Py and SciPy) has been developed by P.Peterson for numerical calculations and experiments (see references).

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Chapter 7
Soft matter physics

Preliminaries

The studies involve statistical topography, percolation and turbulence as the main goals. However, the application the ideas of physics to other fields like econophysics has been extremely useful. Universal notions like fractality or power laws are applicable also in neighbouring fields of physics, like in geophysics, or explaining the behaviour of nanotubes, polymer films, etc.

Statistical topography and percolation

Inverse 6-vertex model and 4-vertex (4V) models have been formulated. These models are suited for the numerical analysis of random self-affine surfaces. In particular, 4V-model has a very high numerical efficiency. Evidences are given that the inverse 6-vertex and 4V models belong to the same universality class. 4V-model has been used to calculate the fractal dimension of a single isoline as a function of the roughness exponent of random self-affine surfaces. New scaling exponent of random self-affine surfaces has been introduced: the fractal dimension of the set of ”oceanic coastlines”. It is shown that for negative roughness exponents, this fractal dimension is equivalent to the fractal dimension of (correlated) percolation clusters. Computer code has been prepared to calculate this exponent as a function of the roughness exponent (Kalda, 2001).

A simple scenario of the formation of geological landscapes has been suggested. It is summarized as follows. Inside a polygon, a random point and direction define the target line for the fault (elevation line). The fault divides the surface into two parts, one of which is elevated, and the other is lowered. The fault follows the aim line as closely as possible in such a way that after the elevation, the modulus of the local gradient remains everywhere below a threshold value. The process is repeated *ad infinitum*. The statistical properties of the resulting gradient-limited surfaces have been analyzed numerically using the super-correlated percolation model. A novel type of scale-invariant behaviour has been predicted: the effective scale-dependent roughness exponent h is a decreasing function of the scale. At the intermediate range of scales, the result h about 0.7 - 0.9 is in a good agreement with the values recorded for geological landscapes. Intriguingly, the roughness exponents of the fracture surfaces vary in the same range (Kalda, 2003).

For random Gaussian surfaces with negative Hurst exponent $H < 0$, the coastlines of oceanic islands have been mapped to the percolation clusters of (correlated) percolation problem. In the case of rough self-affine surfaces ($H > 0$), the fractal dimension of oceanic islands is calculated numerically as a function of the roughness exponent H (using a novel technique of minimizing finite-size effects). For $H = 0$, the result $d = 1.896...$ coincides with the analytic value for the percolation problem (91/48), suggesting a super-universality for the correlated percolation problem. The relationships between various scaling exponents of statistical topography, both for real geological landscapes and for Gaussian surfaces have been analyzed (Kalda, 2008).

The scaling exponents of random surfaces with negative Hurst exponent have been studied numerically by Monte-Carlo simulations. To this end, 1 + 1-dimensional model (4V-model) has been used. In the case of $H = -0{,}75$, the fractal dimension of a single isoline turned out to be $d = 1.563 \pm 0.002$. This is significantly different from the well-known exact result $d=1.75$ for isotropic two-dimensional surfaces. The fractal dimension of the isolines for random surfaces is determined numerically in the range of parameters corresponding to the correlated percolation. It is shown that the universality class of the uncorrelated percolation is narrower than previously thought: the fractal dimension of the isolines is affected by much weaker long-range correlations than the correlation exponent. A novel method of determining scaling exponents from finite-size Monte-Carlo simulation data (developed at CENS) has been tested for reliability and performance (as compared to the classical methods) using analytically well-understood fractal sets (such as percolation clusters and hulls); criteria for choosing the optimal method parameters have been formulated. The results show clearly the superiority of this new technique, which has a wide spectrum of applications in the context of statistical physics of complex systems (Mandre, Kalda, 2011)

A multivariate method of determining scaling exponents from finite-size Monte-Carlo simulation data (developed earlier) has been used to determine the asymptotic correction exponents of the classical percolation problem. Unrivalled numerical accuracy has been achieved by defining and analysing nine different fractal sets (like “bonds", “ends", ”hull", ”lines", “sides", etc.) with equal fractal dimension (Mandre, Kalda, 2013a).

A universal relationship between the scaling exponents has been established describing the time-fluctuations of the intersection size of two moving fractal sets. The scaling laws for the intersections of moving fractal sets have been generalized to the case when the fractal sets evolve in time. The evolution of the fractal sets is described by a dispersion law: the frequency is assumed to be a power law of the wavelength. The fluctuations of the intersection size are described by the Hurst exponent H. The expression relating the exponent H to the fractal dimensions of the fractal sets, and the exponent of the dispersion law. Analytical results are in a good agreement with numerical simulations, and are applicable to the reflection of electromagnetic waves from turbulent sea surfaces (Mandre, Kalda, 2013b)

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Turbulence

The studies into turbulence involve the following problems:

(i) mixing by incompressible fully developed turbulence: anomalous scaling of structure functions, statistical topography of tracer density fields, small-scale anisotropy of passive scalar turbulence, mixing in the case of stationary point source of a tracer, decaying passive scalar turbulence, evolution of droplet size-spectra in warm clouds;

(ii) mixing by smooth chaotic fields: multifractality of the dissipation field, Zipf’s law for the size distribution of tracer blobs, decaying turbulence, and Bachelor’s law – departure from the 1/k power spectrum due to the effect of seed diffusivity or due to decaying/multiplying tracer particles;

(iii) mixing in compressible flows (such as the free-slip surface of turbulent fluids): non-universality of the classical definition of compressibility, modified Richardson's law for the pair dispersion of particles, sticky *vrs* non-sticky tracer particles, fractality of the tracer field, and compressibility effects in the case of sea surface transport;
(iv) dispersion of particles in potential velocity fields (over-damped motion of particles in potential landscapes): the effect of so-called giant pair diffusion (in one-dimensional geometry) and qualitative changes to that phenomenon in two- and three-dimensional geometries;
(v) mixing in quasi-stationary quasi-two-dimensional velocity fields: statistical topography of self-affine Gaussian surfaces, mixing in single scale velocity fields, and mixing in velocity fields with a power-law spectrum.
In general terms, the mixing of a passive scalar Φ in the velocity field *v* is described by the following equation:

$$\frac{\partial \Phi}{\partial t} + \mathbf{v} \cdot \nabla \Phi = k \nabla^2 \Phi + q, \qquad (7.1)$$

where $\mathbf{v}(\mathbf{r},t)$ is in our case a chaotic isotropic single-scale incompressible velocity field, k is diffusivity and q-a source of a passive scalar. In the case of smooth chaotic velocity fields, it is convenient to describe the process using the Fokker-Planck equation for the probability density function σ of the logarithms λ of the tracer gradients (Kalda, 2011)

$$\frac{\partial \lambda}{\partial t} + u \frac{\partial \lambda}{\partial \sigma} = D \frac{\partial^2 \lambda}{\partial \sigma^2}, \qquad (7.2)$$

where u and D are expressed as statistical moments of stretching statistics of the velocity field (Kalda, 2000b).
In case of non-smooth compressible flows the probability density function σ is described by (Kalda, 2007)

$$\frac{\partial \sigma}{\partial t} + u \frac{\partial}{\partial \lambda}\left(f(\sigma,\lambda)\right) = \frac{\partial^2}{\partial \lambda^2}\left(f(\sigma,\lambda)\right), \qquad (7.3)$$

where $f(\sigma,\lambda) = \sigma \exp\left(\lambda(\xi - 2)\right)$, ξ is the smoothness exponent and time t is now renormalized compared to time in Eq. (7.1). The models (7.1), (7.2), (7.3) permit to analyse many interesting cases because turbulent mixing is closely related to statistical topography.

The results are following.
The passive scalar convection by chaotic two-dimensional incompressible flow has been studied. Analytically solvable equations were suggested to describe the evolution of the probability density functions of tracer gradients and power spectra. The multifractal spectrum f(α) of the scalar dissipation field has been calculated. The strict multifractality holds only for small values of α. Stationary and exponentially decaying power spectra of the scalar have been obtained (Kalda, 2000a).
A new multifractal scalar field "harmfulness" quantifying the inhomogeneity of the passive scalar distribution in turbulent flow has been introduced. Assuming technological wastes perform the role of passive scalar, the "harmfulness" is the measure of the environmental damage. It is shown that in the case of passive scalar turbulence, both the multifractality of the "harmfulness" is caused by uneven stretching of fluid elements (Kalda, 2000b).
The k-spectrum of finite-lifetime passive scalar particles has been analysed. It turned out that the spectrum is different for ageing and non-ageing particles. For non-ageing (exponentially decaying) particles, the k-spectrum is a power law. In the opposite case, the differential scaling exponent is a decreasing function of the wave-vector k (Kalda, 2000b).

The model of turbulent diffusion (which has been until now applicable only to two-dimensional flows) is generalized and applied for studying three-dimensional turbulent diffusion. An approach for studying pair dispersion (”Richardson law”) in fully developed turbulence of compressible fluids (e.g. free-slip surface flows) is proposed and analytical expressions for the scaling exponents of the pair-dispersion time-evolution exponents are derived. It is shown that the classical definition of compressibility needs to be modified in order to characterize adequately the time-correlated (non-Kraichnan) flows, and a suitable substitution is suggested. The dispersion depends on the stickiness of the tracer particles and the pair-dispersion exponents for sticky particles are derived (Kalda, 2007)

The aggregation of sticky passive tracer particles in rough compressible velocity fields (which can be realized on the free-slip surface of a turbulent fluid) has been studied. The scaling exponents describing the aggregation are expressed via the compressibility, and the exponent of the turbulent spectrum. A novel description of the passive scalar distribution in chaotic smooth velocity fields has been formulated, which takes into account both the multifractality of the dissipation field, and the power-law size-distribution of the scalar blobs. A new definition of the compressibility of turbulent flows has been devised, based of the dispersion statistics of a pair of passive particles (Kalda, 2007)

The evolution of material lines in nonsmooth turbulent velocity fields has been studied numerically. At the statistically stationary state, the fractal dimension of the material lines appears to be a non-monotonic function of the smoothness exponent of the velocity field. A qualitative explanation of this behaviour is provided. The maximal value of the fractal dimension 1.33 is achieved in the neighbourhood of the Kolmogorov spectrum smoothness exponent (Kalda, Kree).

Fully developed turbulence is known to be highly intermittent, characterized by non-Gaussian statistics and non-vanishing probabilities of extreme events. A novel simple model for the evolution of passive tracers in turbulent flows is developed. Based on that model, an expression for the structure function scaling exponent (which is in a good agreement with the existing numerical and experimental data) is derived, which has revealed the origin of the small-scale anisotropy (Kalda, Morozenko, 2007; 2009)

The origin of the small-scale anisotropy of passive scalar turbulence is studied. The scaling exponent for the third order structure function is derived which is in a good agreement with the experimental data. For fully developed turbulence numerical studies are extended to a broader range of the velocity field smoothness exponents, now including values $\xi \leq 1$.The results show that in the case of passive scalar turbulence, the reconnection of isodensity lines plays a fundamental role: the fractal dimensionalities of the material lines and isodensity lines are clearly different for almost all the values of the smoothness exponent, and are equal for the Kolmogorov value $\xi = 4/3$ only by a coincidence (Kalda, Kree, 2015).

The scaling exponent describing the small-scale anisotropy of passive scalar turbulence is analytically derived; the results are in a good agreement with the simulation results using the one-dimensional turbulent mixing model, as well as with the experimental results.

The rain nucleation process in warm clouds is studied by extending the previously developed “triplet”model of passive scalar mixing in Kolmogorov turbulence. The probability density function of the width of the droplet-size spectrum is derived; this is the first step in explaining the experimentally observed anomalously fast nucleation process. The experimentally observed anomalously fast growth of the width of these spectra is explained theoretically. The scaling exponents of arbitrary-order moments of tracer particle displacement vectors in stationary self-affine two-dimensional velocity fields are derived. The k-spectrum of decaying, aging and growing particles in smooth chaotic flows is derived, addressing, in particular, the problem of plankton patchiness.

The stochastic triplet-map model of turbulent mixing has been also used to analyse the two-dimensional turbulent mixing of a tracer injected from a localised point source. The structure function scaling exponents have been derived as a function of the Hurst exponent of the stream function; the results are verified numerically.

A Fokker-Planck equation describing the evolution of the k-spectrum of passive scalars in compressible flows has been derived analytically. This is an efficient tool for analytical studies of the statistical properties of tracer fields. The breakthrough has been achieved by making a connection between the smallest finite-time Lyapunov exponent of chaotic velocity fields, and transport along the coordinate describing logarithmic increase in tracer density gradient.

A novel concept of finite-time compressibility has been introduced; it has been shown that at the limit of Kraichnan flows, it coincides with the classical compressibilities, and in the case of real, time-correlated flows, it provides more reliable results.

Analytical expressions have been obtained for the finite-time Lyapunov exponents and their variance for chaotic two-dimensional incompressible flows with finite correlation time. This is an important result for understanding the mismatch between the experimental results obtained for real flows, and ideal flows which are delta-correlated in time and for which a wide spectrum of theoretical results exists. The analytical results show a clear departure from the behaviour of ideal flows (Ainsaar, Kalda, 2015).

The mixing of tracers originating from two distant sources has been studied experimentally at the IRPHE laboratory, University of Marseille. This process gives rise to patterns, where some of the patches from different sources merge, but other patches will disperse into the environment without merging. The merging rate can be described via the cross-correlation coefficient as a function of time. Experimental results are in a good agreement with theory (Kree et al. 2013).

Finite-time Lyapunov exponents (FTLE), together with their variance have been expressed for isotropic homogeneous chaotic two-dimensional compressible and incompressible flows with finite correlation time in terms of the velocity gradient tensor statistics. In particular, it has been shown that these exponents depend not only on the dimensionality and compressibility of the flow, but also on the velocity gradient tensor determinant statistics. This result represents a breakthrough in the understanding of the role of time correlations for turbulent mixing.

Majority of theoretical results regarding turbulent mixing are based on the model of ideal flows with zero correlation time. The reasons why such results may fail for real flows have been analyzed and a scheme which makes it possible to match real flows to ideal flows is developed. In particular, the concept of mixing dimension of flows which can take fractional values is introduced. For real incompressible flows, the mixing dimension exceeds the topological dimension; this leads to a local inhomogeneity of mixing — a phenomenon which is not observed for ideal flows and has profound implications, for instance impacting the rate of bimolecular reactions in turbulent flows. It has been shown that finite time-correlations lead to non-universality of mixing as the mixing dimension depends on the statistical properties of the underlying flow.

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Econophysics

The distribution of low-variability periods in the long-term history of stock-market indices (MSCI World, SP500) has been studied. It turned out that similarly to what is observed for the time-series of HRV, the distribution follows a power law (Zipf's law) (Kitt, Kalda, 2004).

The scaling properties of the time series of asset prices and trading volumes of stock markets have been analyzed. It has been shown that similarly to the asset prices, the trading volume data obey multi-scaling length distribution of low-variability periods. In the case of asset prices, such scaling behavior can be used for risk forecasts: the probability of observing next day a large price movement is (super-universally) inversely proportional to the length of the ongoing low-variability period. Finally, a method is devised for a multi-factor scaling analysis (Kitt, 2003).

A super-universal law derived in the context of low-variability period analysis was addressed. Previously, it was shown that just the very presence of a power-law of the length-distribution of low-variability periods leads to a super-universal law that the probability of observing a large movement is inversely proportional to the length of the ongoing low-variability period. This relationship was successfully tested in a number of financial time series. Given the fact that the low-variability periods are closely related to the volatility, this analysis forms a complementary part of risk and volatility analysis (Kitt, Kalda, 2005a; 2005b). The super-universal distribution of low-variability periods has been tested using various financial time-series. The results confirm theoretical expectations and reveal one source of universality in the dynamics of financial markets. It is also useful for the portfolio risk analysis (Kitt, Kalda, 2006; Kitt et al., 2009).

Complex systems have been discussed in the context of economic and business policy and decision making. In particular, it has been argued that small economies have good prospects to gain from the global processes underway, if they can demonstrate flexible production, reliable business ethics and good risk management. For policy making under complexity, following principles are offered: openness and international competition, tolerance and variety of ideas, self-reliability and low dependence on external help. Several applications are elaborated in more detail. The first application demonstrates that small economies have good prospects to gain from the global processes underway, if they can demonstrate production flexibility, reliable business ethics and good risk management. The second application elaborates on and discusses the opportunities and challenges in decision making under complexity from macro and micro economic perspective. Regarding the challenges in

decision making under complexity from macro and micro economic perspective, it has been concluded that the main task for corporate managements is finding and continuously maintaining the balance between short term noise and long term chaos whose attractor includes customers, shareholders and employees (Kitt, 2014).
Based on the wire transfer database of Swedbank, a model of economical network of Estonia has been created. This is world-wide a unique database as it covers dominating majority (ca 80%) of the economy of a single country. The scaling-free and structural properties of this network are studied and its topology, components and behaviours described. It has been shown that this network shares typical structural characteristics known in other complex networks: degree distributions follow a power law, low clustering coefficient and low average shortest path length. Simulations of resiliency of the network against random and targeted attacks of the nodes with two different strategies are performed. Thereby the most vulnerable nodes of the Estonian economy with respect to economic shocks have been found. This finding can be used to develop strategies for increasing economical stability of Estonia (Rendon et al., 2016).
A general overview of studies into econophysics in Estonia is presented by Patriarca et al. (2010).

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Fractality in physics

Scale-invariance phenomenon is studied in the context of geophysics. The model of formation and merger of magmatic batches has been analysed analytically. The size-distribution law of batches has been derived. This is a power law, the scaling exponent of which is either 2/3,5/6, or 1, depending on the model parameters. The analytic results are in agreement with numerical experiment (Kalda).
The statistical topography of polymer films containing PEDT/PSS complex has been studied in cooperation with the TUT Institute of Material Sciences; the anisotropy of the correlated percolation network has been established. A percolation model for describing the conducting properties of these films is developed. A simple lattice model for describing the fragmentation

of drying gel-films is constructed; preliminary results are in a good agreement wit the experiments and reproduce the formation of parallelogram-shaped fragments.
A two-stage lattice model is developed and studied numerically, which explains reasonably well the formation of nanotubes, observed in the experiments at UT. At the first stage, a mesh of craks is formed due to an inhomogeneous contraction of the polymer-bonds of the drying film; at the second stage, the film is released from the substrate, and the film fragments roll into tubes. The cracking of sol-gel films has been modelled and numerically simulated with a variant of spring-block model, which is adapted to describe inhomogeneous drying of films lying on a non-solid substrate. The model results made it possible to understand the dependence of the structure of cracks on the control parameters of the respective experiments. This is an important step towards an optimal design of the sol-gel film cracking experiments, bringing towards industrial applications, such as the production of nanotubes (formed from the film fragments) (Kalda, Jõgi).
Electrical conductivity in thin films of PEDT/PSS complex has been studied using the morphology, created by the Mesodyn-software, and application of percolation theory methods. It has been shown that the strongly anisotropic global conductivity can reasonably well be estimated as the bottleneck-conductivity.
Random walks of point particles on fractals exhibit subdiffusive behaviour, where the anomalous diffusion exponent is smaller than one, and the corresponding random walk dimension is larger than two. This is due to the limited space available in fractal structures. Here, the particles have an orientation which influences their dynamics on fractal structures and there are the dynamical consequences of the interactions between the local surrounding fractal structure and the particle orientation. These interactions can lead to particles becoming temporarily or permanently stuck in parts of the structure. A surprising finding is that the random walk dimension is not affected by the orientation while the diffusion constant shows a variety of interesting and surprising features (Haber et al.,2013;2014)

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Chapter 8
Related fields

Preliminaries
Beside the core groups, several other groups have joined CENS for a certain period:

Geometry, University of Tartu, 1999-2007 (supervisor M.Rahula);
Biomedical engineering, Tallinn UT, 1999-2007 (supervisor H.Hinrikus);
Proactive technologies, Tallinn UT, 2008-2010 (supervisor L.Mõtus);
Nonlinear control theory, 2011-2015 Tallinn UT (supervisor Ü.Kotta);
Optics, University of Tartu, 2011-2015 (supervisor P.Saari).

The studies in these groups were also related to nonlinear problems and/or complexity. Below their result are briefly summarized.

Geometry
Actually the main idea of studies is developing geometric approach to nonlinear problems. It has been shown that any motion along an orbit of a Lie group can be described in an invariant way by a jet structure of mappings. This result allows to study the symmetries of differential equations with the help of the Cartan method which means a certain breakthrough in the studies of differential geometry and differential equations. A generalization of exterior calculus with exterior differential d satisfying $d^N = 0$; $N > 2$ is constructed and studied. The construction of generalization is based on the notion of graded q-differential algebra and non-commutative first order differential calculus.
The higher order cohomologies are studied for exterior calculus, in particular, the differential forms with $d^3 = 0$ on a free associative algebra and reduced quantum plane. For exterior differential their covariant generalization is given that restates the tensorial character of needed transforms. Lie calculus is used for describing the dynamical systems.
The representations of the group GL(2,R) in the space of symmetric (p,0)-forms have been studied. In this case the connection between Veronese maps and operators of Hilbert is shown. This connection allows to give an interpretation of the operators of Hilbert by means of Lie vector fields. The representations of the group GL(n,R) in the space of symmetric vector-valued (p,1)-forms in connection with classification of polynomial dynamic systems are studied.
An approach to the theory of non-linear dynamical systems based on the notion of connection in fiber bundle and on the notion of translation of fiber is developed. Contemporary interpretation of classical tensor analysis based on vector bundles and tangent bundles is offered and possible applications in quantum field theory are described. It is shown objects of connection and curvature in the bundle as operators of type Monge-Ampere arise based on Lychagin theory. Appears that Cartan's forms in infinite jet space, that are Lie derivatives, are φ-connected with corresponding operators in the Spencer schema. In this way the problem of integration arives in the structure of an infinite jet space. By developing theory of connection in the bundle it is shown how, in general case, the morphism of bundles with connections can be developed to automorphism of one bundle and so to movements in the different spaces. Thus, classical notations, such as Lie derivatives of connections and Killins's vector field in Riemann space, obtain a more general interpretation.
It is shown that the adequate geometric apparatus for the topological field theory on a four-dimensional manifold is the theory of superconnections. The Lagrangian of the topological

field theory can be derived from the curvature of a superconnection and the Bianchi identity implies the supersymmetries of the Lagrangian.
The transformation of tensor field in the flow of vector field is described by Lie-Maclaurin series with Lie derivatives for coefficients. As the tensor field is given in natural or anholonomic basis the calculations begin with derivation formulae and the matrix C determined through the components of vector field and object of anholonomity. If the tensor field is invariant in the flow then its components satisfy a linear ordinary differential equation corresponding to Hamilton-Cayley formula. And on the contrary, if the components are invariant then the tensor field itself with Lie derivatives satisfies a similar but dual differential equation. Both cases are described with the help of proper values (in the knot points of a network on complex plane).
A notion of a Z_N-connection, where N is any integer equal or greater than 2, which can be viewed as a generalization of a notion of Z_2-connection or superconnection is proposed. The algebraic approach to the theory of connections to give the definition of a Z_N-connection is used for exploring its structure.
The concept of a q-connection and graded q-connection, which can be viewed as generalizations of connection and superconnection, are proposed and studied by means of a module over a graded q-differential algebra, where q is a primitive Nth root of unity. It is proved that the curvature of a (graded) q-connection satisfies the Bianchi identity, and the components of the curvature are expressed in terms of the coefficients of a q-connection provided that the module is a finitely generated free module.
The geometrical approach to the theory of differential prolongations in jet spaces (total differential operators, contact forms, their symmetries and invariants) has been developed, using global analysis and Lie-Cartan methods.

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Biomedical Engineering

The main areas of studies are the analysis of biological signals and brain research.

For biosignals interpretations, the different nonlinear L- filtering, Stack filtering and Linear-WOS (Weighted Order Statistic) Hybrid filtering have been applied for the analysis ECG and EEG signals.The general ideas behind the mentioned filtering algorithms, most useful subclasses of the filters and their application in physiological signal processing were analysed. It is shown that depression of alpha-waves in human EEG takes place after modulated low-level microwave exposure. The simple coherent optical method based on nonlinear effects in laser active media was developed for pulse wave shape and velocity measurement. It is proved experimentally that the coherence length of light in the case of selfmixing is much longer than for traditional interferometry.

EEG monitoring during cardiac surgery has carried on for representing the data in a compact way to study the changes in signal behaviour in different phases of surgery.

In order to reveal the possible correlation between the level of myocardial electrical instability assessed at Holter monitoring and certain ECG parameters characterizing ventricular repolarization 24-hours ECG recordings were analyzed. Thus, the results of our study demonstrate that several parameters characterizing ventricular repolarization phase on surface ECG appear to have strong correlation with the level of myocardial electrical instability assessed at Holter monitoring. The determination of the clinical significance of these parameters requires further investigations. The utilization of the study results - non-invasive analysis of temporal ventricular repolarization heterogeneity and certain aspects of T-wave morphology - provides an important tool for sudden death risk stratification and assessment of antiarrhythmic drug treatment benefits/drawbacks in patients.

The pulse wave transit time has been shown to be a useful parameter to estimate cardiovascular instability. A method was developed for estimating the variation of blood pressure using pulse wave transit time, namely the delay between the heart electrical signal and moment of reaching of pulse wave to a selected peripheral site. The method is based on presumption that there is a singular relationship between the pulse wave delay and the variation of blood pressure in artery for a given person.The impact of the pre-ejection period and instability of the vessels' parameters on the accuracy of the estimation of the arterial blood pressure using the time delay between heart electrical signal and pulse wave arrival to a selected peripheral site was analysed. The physiological signals (such as ECG, fingertip photoplethysmograph (PPG), and invasive arterial blood pressure (IBP) as a reference signal) from Physionet Mimic database were used. Measurements of pulse wave shape were performed in normal conditions and with applying Cold Pressure Test (CPT). The results showed that pulse wave shape was significantly changed with increasing of blood pressure. The CPT caused statistically significant changes in pulse wave shape in the case of patients with normal blood pressure and did not induced significant change for the patients with hypertension. The preliminary results show that it is possible to use pulse wave shape analysis for noninvasive indirect estimation of elastic properties of blood vessels.

The bispectral analyses, coherence analyses and fractal dimension analyses are used to evaluate the effect of low-level microwave radiation on human EEG alpha and theta rhythms. Parallel statistical evaluation of the changes in the EEG signal energy level was performed. The parameters that calculated from the EEG bispectrum could not detect the influence of the microwave radiation. Coherence and fractal dimension analyses showed the effect in case of some individuals. In the majority of cases, photic stimulation caused changes in the EEG energy level in the occipital and microwave stimulation in the frontal region.

The studies into brain electrical oscillations under influence of external periodic stressors or depressive disorders. The applied methods were based on detection of specific features in the resting electroencephalographic (EEG) signal and visual event-related potential (ERP). The

comparison of the traditional spectral analysis and a new scale-invariant method - analysis of the length distribution of low variability periods (LDLVP) – to distinguish between the EEG signals with and without a weak stressor - a low-level modulated microwave field - was done.
The dynamics of the low-level microwave radiation effect on human EEG alpha and theta rhythms was analyzed. The experimental results demonstrated that microwave stimulation effects became apparent starting from the third stimulation cycle.
The comparison of two instantaneous frequency (IF) estimates of spindles during propofol anesthesia at burst-supression level, based on Wigner-Ville distribution (WVD) and optimized generalized marginals Choi-Williams (GMCW) distribution was done. The test showed that both methods yielded similar results.
Modulated microwave radiation at low non-thermal level of field power density can affect human central nervous system in a sensible way, for example, on discrimination of changes, produced by low-level microwave exposure in intensity and time variability of the human EEG at rest. The effect of low-level microwave exposure is stronger on EEG beta rhythm in temporal and parietal regions of the human brain. A possible origin of interaction mechanism of microwave radiation with nervous system - quasi-thermal field effect - was investigated. Analysis of the experimental data shows that: 1) statistically significant changes in EEG rhythms depend on modulation frequency of the microwave field; 2) microwave stimulation causes an increase of the EEG energy level; 3) the effect is most intense at beta1 rhythm and higher modulation frequencies. Individual sensitivity to modulated low-level microwave exposure effects on human EEG theta, alpha and beta rhythms was evaluated. The effect of microwaves modulated at different frequencies on human EEG rhythms was evaluated.
Two new methods for EEG analysis were developed and evaluated in the same database. A nonlinear method of scaling analysis of the EEG signal based on the length distribution of low variability periods (LDLVP) was adapted for EEG analysis. The method of modulation with further integration of differences in energy from EEG segments with and without microwave exposure (IDE) was developed and applied for EEG signals.

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Proactive technologies

The conceptual basis for the future research into proactive technologies has been elaborated which focuses on practical difficulties and paradoxes hindering rapid dissemination of pervasive computing systems, with special emphasis on integration and networking of component-based stand-alone systems caused phenomena. The key to resolution of observed difficulties lies in better understanding the essence of the underlying computational processes, in providing the computational processes with adequate ambient information, and in enhancing self-X capabilities of the synthesised systems.

The research is built on the following statements:

- the new pervasive computing systems violate, strictly speaking, the axioms and restrictions imposed by Turing computing paradigm, hence the Turing computing paradigmbased models provide too approximate description of systems' behaviour;
- the previous statement matches with the observations pointed out at the events of Grand Challenges for Computing Research, that leads us to search a solution within non-classical computation paradigms - in the present case within situation-aware interaction centred models of computation;
- simultaneous and interacting research threads (models of computation and proactive models of applications, proactivity, autonomy and situation-awareness of systems, and technological platforms for implementation) – as practiced in the lab – have turned out to be fruitful and symbiotic, although not very easy to coordinate and to synchronise;
- systems comprising of autonomous (and may be proactive) components with dynamic structure of interactions have secured their position among novel computer applications and the related impacts and unsolved problems cannot be overlooked;
- autonomy and proactivity in artificial (as well as natural) systems assume the existence of dynamic structure of inter-component interactions and inevitably cause the level of emergent behaviour that cannot any more be neglected;
- for (partial) control of emergent behaviour one needs to understand the essence of the underlying computations - Turing machine paradigm cannot explain the computations in networked pervasive computing systems with sufficient details, and completely neglects many essential features;
- properties of the pilot multi-stream interaction machine (as a case study of a tool for reasoning about interaction-centred computation) look promising for detection of many dynamically emerging features and enables to embed into the computing system instruments (e.g. mediated interactions) for partially controlling the emergent behaviour;
- the experiments with detection and partial control of the emergent behaviour in artificial systems will provide better insight into the essence of emergent behaviour observed in the natural and/or social systems.

The research included the analysis of adaptation and self-organisation, architectural control in virtual organisations, simulations in ad hoc networks and simulators, acquisition and preprocessing of the situational information, situation awareness of vehicles, nonlinear discrete-time systems, computational intelligence methods for process control.

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Remark: The results have been mostly published in Conference Proceedings.

Nonlinear Control Theory

Attention is focused on novel algebraic methods and symbolic software tools for solving fundamental problems in nonlinear control. The approach is based on theory of non-commutative polynomial rings, linear algebra over the field of meromorphic functions, lattice theory and time-scale analysis. As a result, one can address general continuous- and discrete-time systems, including non-smooth systems and systems, defined on more general time scales. The software is of open access and used by researchers and graduate students world-wide.

The reduction and realisation problems are solved for nonlinear control systems applying the theory of non-commutative polynomials. The main advantage of polynomial approach is ‘computability’; the theoretical results are complemented by explicit formulas yielding a short program code in Mathematica-based symbolic software. Computational aspects of realisation of a set of higher order nonlinear input/output equations in the state space form are analysed. Instead of the algorithmic solutions, provided in earlier studies, the explicit formulas, based either on the concept of adjoint polynomials or on division of polynomials, are obtained. Using the nonlinear realization theory, necessary and sufficient conditions are provided for linear parameter-varying input-output equations to be transformable into a state-space form with static dependence on the so-called scheduling parameter.

The inversive differential ring, associated with a nonlinear control system, defined on a non-homogeneous but regular time scale is constructed and equipped with three operators (delta- and nabla-derivatives and forward shift operator) with their properties determined. The developed formalism unifies/extends those for continuous- and discrete-time systems.

The observable space of the nonlinear system is analysed using the time scale formalism. This allows formulating the conjecture about the possible nonintegrability of the observable space in the discrete-time case. It has been shown that for the special subclass of reversible polynomial discrete-time systems the observable space is always integrable. The simple necessary and sufficient conditions, allowing transforming the nonlinear discrete-time control

system into the extended observer form, are provided. The solvability conditions are formulated in terms of certain partial derivatives and due to the matrix representation they can be checked almost by direct inspection.
In order to address the disturbance decoupling problem by dynamic measurement feedback for discrete-time systems, the algebraic approach is applied. This approach, based on lattice theory, allows the system description also depend on non-differentiable functions. A necessary and sufficient condition is given in terms of controlled and (h,f)-invariant functions. Also, algorithms are derived which find the invariant functions and the required feedback. In order to address the disturbance decoupling problem by the dynamic measurement feedback for discrete-time systems, the algebraic approach is applied. This approach, called the algebra of functions allows the system description also depend on non-differentiable functions. A necessary and sufficient condition is given in terms of controlled and (h,f)-invariant functions. Also, algorithms are derived which find the invariant functions and the required feedback. Using the nonlinear realization theory, necessary and sufficient conditions are provided for linear parameter-varying input-output equations to be transformable into a state-space form with static dependence on the so-called scheduling parameter. The observable space of the nonlinear system is analysed using the time scale formalism. This allows formulating the conjecture about the possible nonintegrability of the observable space in the discrete-time case.

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Optics

Formation of localized light bullets like Bessel-X waves and Airy-Bessel pulses by means of diffractive and refractive optical elements is theoretically analysed and experimentally realized. The boundary diffraction wave theory is generalized and adopted for Gaussian pulses. Predictions of the theory are verified experimentally by measuring diffraction of ultrashort optical pulses on various screens. Nonlinear second-harmonic generation with laser

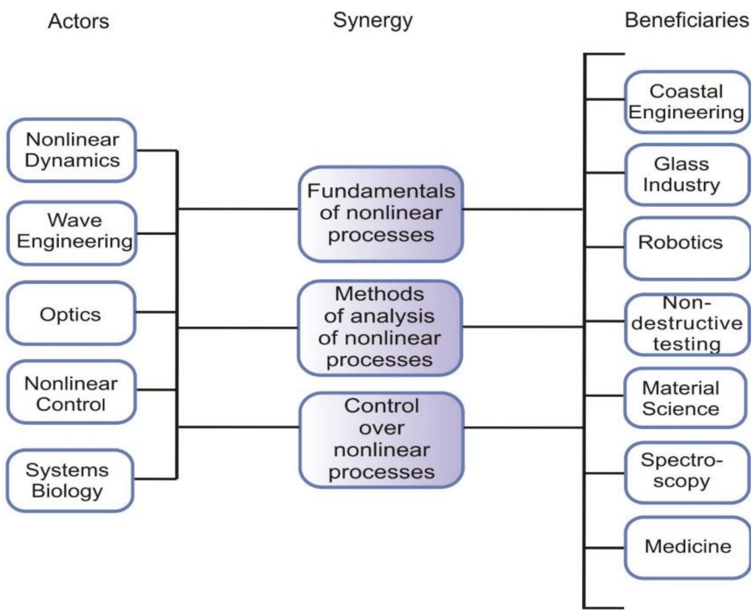
beams transformed by internal conical refraction in a biaxial crystal is studied, and transformation of vortex Laguerre-Gauss laser beams by conical refraction several specific second-harmonic beam profiles are demonstrated. For the first time temporal focusing of ultrashort pulsed Bessel beams into Airy-Bessel light bullets by a circular diffraction grating is verified. By theoretical studies and direct measurements with ultrahigh temporal and spatial resolution it is shown that a setup based on a spatial light modulator with an imprinted wrapped cubic phase shapes femtosecond pulses into curvilinearly propagating Airy pulses of type II, whereas a set-up based on a custom-made refractive element with a cubic phase surface profile produces type IV nondispersing Airy pulses of exceptional high quality.
A set-up for spatiotemporal measurement of ultrashort impulse responses of optical systemswith up to 5-femtosecond temporal resolution is accomplished by using a supercontinuum laser source and photonic crystal fibres. For the experiments an ultra-broadband version of spectral interferometer is constructed which allows to record impulse responses with ultrahigh (single-cycle) temporal and micrometer-range spatial resolution. In 2014, the Optics Group as the whole Institute of Physics of the University of Tartu moved into the new building. The conditions to carry out experimental research in the fields of physical, nonlinear and ultrafast optics became much better.

References

P. Piksarv, P. Bowlan, M. Lõhmus, H. Valtna-Lukner, R. Trebino, P. Saari. Diffraction of ultrashort Gaussian pulses within the framework of boundary diffraction wave theory. J. Optics, 2012, 14, 1, paper 015701.
P. Piksarv, H. Valtna-Lukner, A. Valdmann, M. Lõhmus, R. Matt, P. Saari. Temporal focusing of ultrashort pulsed Bessel beams into Airy-Bessel light bullets. Opt. Express, 2012, 20, 17720-17729.
P. Piksarv, A. Valdmann, H. Valtna-Lukner, R. Matt, P. Saari. Spatiotemporal characterization of ultrabroadband Airy pulses. Opt. Lett., 2013, 38, 7, 1143-1145.
V. Peet. The far-field structure of Gaussian light beams transformed by internal conical refraction in a biaxial crystal. Opt. Comm., 2013, 311, 150-155.
A. Valdmann, P. Piksarv, H. Valtna-Lukner, P. Saari. Realization of laterally nondispersing ultrabroadband Airy pulses. Optics Lett., 2014, 39, 1977.

Chapter 9
Summary – what is all that about?

In Chapter 1 the main ideology and keywords of CENS activities have been presented and Chapter 2 described the prehistory of studies and research interests. Chapters 3-8 collected the research results over years 1999-2015. In what follows, the summary of activities is presented and analysed. The main question is – what is all that about and whether the pieces form a much larger picture or pattern. During the first decade of CENS such a question was asked not very often but during the last five years, the attention was turned more and more towards general problems and generalizations.
The core groups over 1999-2015 were from the Institute of Cybernetics – Nonlinear Dynamics (waves in solids, soft matter physics, photoelasticity), Wave Engineering (waves on sea and coastal engineering), and Systems Biology (cell energetics). However, the other groups of CENS, although being involved not over the full period, very clearly followed the general ideas of complexity, whether in proactive technology where attention was more to human-computer interaction, or geometric methods for nonlinear problems, or in biomedical engineering where signals from human body were analysed, or control theory where methods for control of nonlinear processes were elaborated, or optics where the similarity of mathematical models to those in solids and fluids is obvious. So the main keywords like nonlinearity and interactions resulted in describing complex phenomena, all embedded into the context of interdisciplinarity.
First let us show the actors and beneficiaries of CENS in 2011-2015.



The idea is extracting the essentials from complicated models and observations but not forgetting the whole. Indeed, the focus on **nonlinear dynamical processes** with due attention to accompanying effects like dispersion, dissipation and diffusion is kept in all the studies. In these studies, the crucial issue is the analysis of interactions: wave-wave type, wave-field type, wave-internal structure type and their combinations in order to understand the physical mechanisms which govern the processes, let them be in solids, fluids or tissues. The nonlinear mathematical models, as a rule, are based on the theory of continua. This is the joint basis and often leads to similar basic governing equations. From the results described above, it is clearly seen that conventional theories must often be improved or modified in order to grasp physical phenomena with a proper accuracy. As far as the models are inherently nonlinear, the numerical methods must also be modified like for simulation of waves in solids, for unification of concepts of discrete and continuous time, for diffusion problems, etc. In this context, the nonlinear control theory is of importance.

A striking feature for waves in solids and waves in fluids is the conceptual similarity of models which stresses the interaction between the constituents. Leaving aside the standard balance of momentum as a basis for equations of motion in continua, there is a possibility of interpreting the interactive forces and/or fields in a similar way for both cases. In solids, it has been shown that on the material manifold, the governing wave equation based on the balance of the canonical (material) momentum reads:

$$\left.\frac{\partial \mathbf{P}}{\partial t}\right|_X + \mathrm{Div}_R \mathbf{b} = \mathbf{f}^{\mathrm{inh}} + \mathbf{f}^{\mathrm{ext}} + \mathbf{f}^{\mathrm{int}}, \tag{9.1}$$

where $\mathbf{P}$ is the material momentum (pseudomomentum), $\mathbf{b}$ is the material Eshelby stress, and $\mathbf{f}^{inh}, \mathbf{f}^{ext}, \mathbf{f}^{\mathrm{int}}$ are the material inhomogeneity force, the material external (body) force, and the material internal force, respectively.

For water surface waves, the slow energy exchange within resonance quartets is described by the so-called kinetic equation, which today is the core of spectral wave prediction models:

$$\frac{\partial N_1}{\partial t} + \nabla \cdot \left(\mathbf{c}_g N_1\right) = S_{NL} + S_{diss} + S_{In}. \tag{9.2}$$

Here $N(\mathbf{k})$ is the wave action spectral density, $\mathbf{c}_{\mathrm{g}}$ is the group velocity, the so-called interaction integral

$$\begin{aligned} S_{NL} = \int \left|T_{1234}\right|^2 &\left[N_3 N_4 \left(N_1 + N_2\right) - N_1 N_2 \left(N_3 + N_4\right)\right] \\ &\times \delta^2(\Delta \mathbf{k})\delta(\Delta\omega) d\mathbf{k}_{234}, \end{aligned} \tag{9.3}$$

usually characteristic of the Boltzmann equation (where it describes collisions between particles), integrates the contribution from nonlinear interactions into changes in the wave fields, T_{1234} is the interaction coefficient, $\Delta\mathbf{k} = \mathbf{0}$, $\Delta\omega = 0$ are the resonance conditions for the wave vectors and angular frequencies, respectively, $S_{\mathrm{diss}} = -\gamma_D(\mathbf{k})N(\mathbf{k})$ reflects dissipation of wave energy due to different reasons and $S_{\mathrm{In}} = \beta(\mathbf{k})N(\mathbf{k})$ expresses the wind input to the wave systems.

In Eqs (9.1) and (9.2) the forcing term (the r.h.s.) for the governing equation reflects the interaction features associated with changes in certain properties of the counterparts. This way of description of processes is generic and universal almost everywhere in our world when nonlinearity gives birth to situations where the whole has new features compared to simple addition of the counterparts. True, the variables are different but the idea is the same: an action is driven by a combination of (possibly a continuum of) several forces which describe the complicated nature of constituents or processes. From the viewpoint of complexity science, this is essential for emerging macroprocesses.

The next question is about the cooperation between the groups which can be illustrated by the matrix of cooperation:

Added value given / Added value obtained	Nonlinear Dynamics	Wave Engineering	Optics	Nonlinear Control	Systems Biology
Nonlinear Dynamics		methods, 2D soliton theory	optical wavebeams	control over steering solitons	internal variables in biophysics
Wave Engineering	methods, turbulent mixing, 3D images		models of dispersive waves	control over long waves	
Optics	solitons, laser-based tomography	solitons, wave packets		growth of nanotubes, control of localised waves	spectroscopy
Nonlinear Control	control over wave processes	control in environmental processes	control over wave processes		control in real time of single cell
Systems Biology	thermodynamics in physiology, 3D images		optical microscopy	control of cell energetics	

Many results reflect the **synergy** between various disciplines: interaction of solitons in solids and fluids, contraction of muscles in tissue mechanics and links to internal variables, formation of patches of pollution on the sea using the knowledge from currents and soft matter physics, formation of diffraction patterns and localized waves which are characteristic in optics but also in solids, wave propagation in channels (fluids) and in laminated materials (solids), modelling of social systems for which the methods of soft matter physics are used (graph theory for modelling fiscal transactions), modelling mechanical waves in biomembranes combining physiology and mechanics of solids. Such studies will be continued. It is obvious that due to interactions new patterns of waves or fields emerge which are clear signs of complexity. Beside these specific results, some published papers are directly related to general complexity problems in engineering and natural sciences. In addition, the algorithms for nonlinear control theory are derived paying attention to various timescales and unifying the discrete and continuous time. This is important for understanding real dynamical processes.
Certainly a question arises: how all the activities of CENS are seen by peers. The International Advisory Board – IAB (see Annex 3) has concluded in 2015:

„All contributing research groups obtained first class results of fundamental importance as well as practical impact. Some groups are among the scientific leaders internationally in the field of nonlinear systems. In their essential parts, the studied topics address the structure and dynamics of nonlinear dynamic processes in natural and artificial systems. The universality of the mathematical laws in the relevant mechanisms was clearly demonstrated. In particular, the results cover nonlinear wave phenomena in structured solids, ocean scenarios, complex biological molecular structures, and structured broadband optical waves and wave-packets. Furthermore, basic mathematical investigations in control theory were presented. The analytical and numerical mathematical tools represent a high standard. Selected topics like ocean dynamics in the Baltic Sea, heart medical studies or glass photoelasticity, to give only a few examples, are obviously of significant potential economic impact. On a fundamental level,

the intriguing analogies between many kinds of wave and complexity effects might be of general interest. They open not only perspectives for novel engineering solutions but insight into the working principles of nature as well.
In some way we should apply the theory of complex systems to the Centre itself. The Centre produces an outcome that is greater than the sum of the parts - the parts here being the individual researchers who could act on their own but see the benefit of mutual activities and promote the sharing of best practice.“

As for recommendations, the IAB notes:
„Among the new noticeable strengths we note the proposal of moving fractal sets and associated scaling laws, the explicit presentation of a general mechanism of single-cell bioenergetics, the method of integrated nonlinear photo-elastic tomography, and the all round approach to waves at sea, wave climatology, pollution, and coastal protection. Many of these subjects are not only truly scientific but they also strongly contribute to socio-economical preoccupations. The field of optics further extends to measuring techniques, plasmons on surfaces and other areas of interest where one has to expect strong synergetic effects and the design of technical devices. In particular CENS has a unique selling point in that engineers and applied mathematicians are brought together with a wide view on the field ranging from important research in control theory to more applied topics around sea waves. The Centre can (beside research) provide an environment for informal exchange of ideas and concepts which are beneficial but often not actually realized in actual publication-producing collaborations.“

Beside the research results, CENS has paid a lot of attention to popularization and teaching (see Annex) in order to disseminate the ideas of complexity and interdisciplinarity.
The science-popular topics involve freak waves (T.Soomere), complexity (J.Engelbrecht, R.Kitt), beauty of dynamics (J.Engelbrecht), coastal engineering (T.Soomere), optical phenomena (P.Saari), networks (J.Engelbrecht), just to name a few. Several results have been described in Encyclopedia:

Encyclopedia of Thermal Stresses (ed. by R.B.Hetnarski), Springer, 2014, pp. 3673-3682:
H.Aben et al., Photoelasticity for the measurements of thermal residual stresses in glass.
Encyclopedia of Complexity and Systems Science (ed. by R.A. Meyers), Springer 2009, pp.8479-8504:
T.Soomere, Solitons interactions.

Teaching in the Tallinn Unversity of Technology and the University of Tartu covers courses in continuum mechanics (A.Salupere), coastal engineering (T.Soomere), quantum mechanics (P.Saari), nonlinear dynamics (J.Engelbrecht), electrodynamics (J.Kalda) mathematical modelling (J.Engelbrecht), etc. J.Kalda has supervised Estonian team for the International Physics Olympiads more than a decade. The fellows have supported translations of the science-popular literature (see Annex). The ideas on science and complexity are represented at the Academic Council of the State President (J.Engelbrecht, until 2011; P.Saari, until 2016); Robert Kitt, 2012-2016).

How we estimate the activities ourselves? Collecting these results, it seems that CENS has been pretty active over 1999-2015. The research will go on because there are many unanswered questions and new challenges in wave dynamics, bioenergetics, optics, microstructured materials, turbulence, etc – all entwined into complexity canvas. This area, as

described above, is wide and needs future studies. The creative environment for research and young people support the perspectives in the future.
It is not by chance that Centre for Nonlinear Studies has an acronym CENS which can be also understood as Complexity in Engineering and Natural Sciences (see the list of Working Groups). This is clearly related to the outcome of the research and also mentioned by peers.

However, one cannot settle down because *„If you want to get somewhere else, you must run at least twice as fast as that“*, like the Queen said to Alice. The author of this Report will pass the baton over to younger colleagues. It has been a pure pleasure to work in CENS and supervise its activities.

General references on CENS activities

CENS Highlights 2003-2007. Tallinn, CENS, 2007.
J.Engelbrecht, A.Berezovski, T.Soomere, Highlights in the research into complexity of nonlinear waves. Proc. Estonian Acad. Sci., 2010, 59, 2, 61-65.
J.Engelbrecht. Complexity in engineering and natural sciences. Proc. Estonian Acad. Sci., 2015, 64, 3, 249-255.
CENS Highlights 2011-2015. Tallinn, CENS, 2015.

ANNEXES

1. Ideology of Research in CENS
2. Groups in CENS
3. International Advisory Board of CENS
4. CENS staff
5. Promoted PhD's
6. Essential publications 1999-2015
7. Selected invited lectures
8. Awards
9. Geography of visitors
10. Geography of visits
11. Essential meetings organized
12. Essential international grants

Annex 1
Ideology of research in CENS

From
physics & mathematics & biology

over
continuum mechanics & hydrodynamics & optics & physiology

to
waves & fields

describing
wave profiles and structures, coherence phase transition, thermodynamics,
dispersion, diffusion,

in processes of
coupling of physical effects, balance of physical effects, formation of structures,
energy (re)distribution, turbulent mixing, dynamics of time series

applied in
mechanics, material science, marine science and coastal engineering,
systems biology,
stress analysis and non-destructive testing, econophysics

Annex 2
Groups in CENS

Head of CENS – J.Engelbrecht
Nonlinear dynamics, 1999-2015 (supervisors J.Engelbrecht until 2014, J.Kalda)
including Photoelasticity (supervisors H.Aben, J.Anton)
Soft matter physics (supervisor J.Kalda)
Wave engineering, 1999-2015 (supervisor T.Soomere)
structural unit since 2008
Systems biology, 1999-2015 (supervisor M.Vendelin)
structural unit since 2007
Geometry, 1999-2007 (supervisor M.Rahula)
Biomedical engineering, 1999-2007 (supervisor H.Hinrikus)
Proactive technologies, 2008-2010 (supervisor L.Mõtus)
Nonlinear control theory, 2011-2015 (supervisor Ü.Kotta)
Optics, 2011-2015 (supervisor P.Saari)

Annex 3
International Advisory Board of CENS

Prof. Josef Ballmann (Aachen), 1999-2006
Prof. David G. Crighton (Cambridge), 1999-2000
Prof. Bengt Lundberg (Uppsala), 1999-2006
Prof. Erik van Groesen (Enschede), 1999-2006
Dr. Andras Szekeres (Budapest), 1999-2006
Prof. Roger Grimshaw (Loughborough), 2006-2015
Prof. Gérard A.Maugin (Paris), 1999-2015
Prof. H.Keith Moffatt (Cambridge) 2006-2011
Prof. Grégoire Nicolis (Brussels) 2006-2011
Prof. Franco Pastrone (Turin), 2006-2015
Prof. Valdur Saks (Grenoble/Tallinn), 2006-2011
Prof. Gábor Stépán (Budapest), 2006-2015
Prof. Dick van Campen (Eindhoven), 2006-2011
Prof. Frank Allgöwer (Stuttgart), 2011-2015
Prof. Steven Bishop (London), 2011-2015
Dr. Rüdiger Grunwald (Berlin), 2011-2015
Prof. Gerhard Holzapfel (Graz), 2011-2015

Annex 4
CENS staff

Aben, Hillar; Abramov, Viktor; Ainola, Leo; Ainsaar, Siim; Anier, Andres; Anton, Johan; Astrov, Igor; Avanessov, Sergei.
Bachmann, Maie; Bazunova, Nadezhda; Belikov, Juri; Berezovski , Arkadi; Berezovski, Mihhail; Birkedal, Rikke; Branovets, Jelena; Braunbrück, Andres.
Delpeche-Ellmann, Nicole; Didenkulova, Irina.
Eelsalu, Maris; Engelbrecht, Jüri; Eik, Marika; Errapart, Andrei.
Ferenets, Rain; Fridolin, Ivo.
Giudici, Andrea.
Herrmann, Heiko; Hinrikus, Hiie; Hlimonenko, Irina.
Ilison, Lauri; Ilison, Olari; Illaste, Ardo.
Janno, Jaan, Jepihhina, Jelena.
Kalda, Jaan; Kalda Mari; Kaldmäe, Arvo; Karo, Jaanus; Kaparin Vadim; Karai, Deniss; Kartau, Katri; Kartofelev, Dmitri; Kask, Andres; Kelpšaite, Loreta; Kitt, Robert; Kongas, Olav; Kotta, Palle; Kotta, Ülle; Kurennoy, Dmitry; Kutser, Mati; Kääramees, Marko.
Laasmaa , Martin; Lapimaa, Triin; Lass, Jaanus; Lipping, Tarmo; Lints Martin; Lints, Taivo.
Mandre, Indrek; Meigas, Kalju; Mullari, Tanel; Mõtus, Leo.
Nikolkina, Irina; Nurges, Ülo; Nõmm, Sven.
Parts, Maie; Pastorelli, Emiliano; Pedai, Andrus; Peet, Viktor; Andrus; Peets, Tanel;
Peterson, Pearu; Pindsoo, Katri; Preden, Jürgo-Sören; Piksarv, Peeter.
Quak, Ewald.
Rahula; Maido; Ramay, Hena; Randrüüt, Merle; Rannat, Kalev; Ravasoo, Arvi; Retšnoi,Vitali; Riid, Andri; Rodin, Artem; Rodina, Anastassia; Rubljova, Jekaterina; Räämet, Andrus.
Saari, Peeter; Salupere, Andrus, Sanko, Jelena; Schryer, David; Šeletski, Anna; Selg, Matti; Sepp, Mervi; Sertakov, Ivan; Srinivasan, Seshadri; Sillat, Tarvo; Simson, Päivo; Sokolova (Karro), Niina; Soomere, Tarmo; Stulov, Anatoli; Säkki, Maksim; Zaitseva-Pärnaste, Inga.
Tamm Kert; Torsvik, Tomas; Tomson, Ruth; Treštšalov, Aleksei; Tseluiko, Dmitri; Tuulik, Viiu; Tõnso, Maris.
Vain, Jüri; Valtna-Lukner, Heli; Viidebaum, Mikk; Viikmäe, Bert; Viška Maija; Värv, Rolf.

Main contributors from Estonia
Elken, Jüri; Erm, Ants; Ernits, Juhan-Peep.
Keevallik, Sirje.
Patriarca , Marco; Petlenkov , Eduard.
Saks, Valdur; Seppet, Enn; Soesoo, Alvar.
Tammet, Tanel; Tepljakov, Aleksei.
Vahisalu, Rein.

Main contributors from abroad over 1999-2015

Bartosiewicz, Zbigniew (Bialystok University of Technology**,** Poland); Beard, Daniel (University of Michigan, USA); Beraud, Natalie (INSERM, France); Bishop, Steven R. (University College London, UK); Braun, Manfred (Universität Duisburg-Essen, Germany).

Casagrande, Daniele (Trento University, Italy); Cermelli Paolo (Torino University, Italy); Choi, Byung H. (University of California, USA); Ciulkin, Monika (Bialystok University of Technology, Poland).

Grimshaw, Roger (Loughborough University, UK); Guerrero, Karen (Joseph Fourier University, Grenoble, France).

Dos Santos, Serge (INSA Centre Val de Loire, France).

Halás, Miroslav(Slovak University of Technology, Bratislava, Slovak Republic); Healy, Terry (Waikato University, New Zealand); Hoffman, Karl-Heinz (TU Chemnitz, Germany).

Kawano, Yu (Kyoto University, Japan); Kolman , Radek (Institute of Thermomechanics AS CR, Prague, Czech Republic); Kurkin, Andrey (Nizhny Novgorod State Technical University, Russia).

Lochegnies, Dominique (University of Lille Nord de France); Lorenzi, Alfredo (Milano University, Italy).

Maugin, Gérard (Université Pierre et Marie Curie, Paris, France); Moog, Claude (Institut de Recherche en Communications et Cybernetique de Nantes); Muschik, Wolfgang (Technische Universität Berlin, Germany); Myrberg, Kai (SYKE/Marine Research Centre, Finland).

Papenfuss, Christina (Hochschule für Technik und Wirtschaft Berlin, Germany); Parnell, Kevin (James Cook University, Australia); Pastrone, Franco (Torino University, Italy); Pawluszewicz, Ewa (Bialystoki University of Technology, Poland); Pelinovski, Efim (Institute of Applied Physics, Russian Acad. Sci., Nijni Novgorod, Russia); Plešek, Jiři (Institute of Thermomechanics AS CR, Prague, Czech Republic); Puttonen, Jari (Aalto University, Finland).

Rehák, Branislav (Institute of Information Theory and Automation, AS CR, Prague, Czech Republic).

Schlattner, Uwe (University Joseph Fourier, France); Sheu, Shey-Shing (Thomas Jefferson University, Philadelphia, USA); Shumsky, Alexey (Institute for Marine Technology Problems, Far Eastern Branch of the Russian Academy of Sciences, Vladivostok, Russia); Szekeres, Andras (Budapest University of Technology and Economics, Hungary).

Zhirabok Alexey (Far Eastern Federal University,Vladivostok, Russia); Zinober, Alan (University of Sheffield, UK); Ziming Liu (San Jose State University, USA).

van Groesen, Erik (Brenny), (University of Twente, Netherlands); Ván, Peter (Institute for Particle and Nuclear Physics, Wigner Research Centre for Physics, HAS Budapest, Hungary); von Hertzen, Raimo (formerly Aalto University, Finland).

Wyrwas Malgorzata (Bialystok University of Technology**,** Poland).

Annex 5
Promoted PhD's

2001: M.Vendelin, P.Peterson
2002: T.Tseluiko, J.Lass
2003: I.Fridolin
2005: A.Braunbrück, R.Kitt, O.Ilison, M.Säkki, V.Retšnoi
2007: K.Rannat, R.Ferenets
2008: J.Anton, I.Didenkulova
2009: L.Ilison, D.Kurennoy, A.Kask, L. Kelpšaite, A.Kull
2010: M.Berezovski, M.Randrüüt, A.Räämet, J.-S. Preden
2011: T.Peets, K.Tamm
2012: A.Errapart, O.Kurkina, J.Belikov, O.Kurkina, D. Schryer, M.Kääramees
2013: I.Didenkulova (DSc), M.Sepp, P.Piksarv, V.Kaparin, I.Zaitseva-Pärnaste
2014: D.Kartofelev, B.Viikmäe, M.Viška, N.Delpeche-Ellmann, M.Eik
2015: A.Giudici, A.Rodin, E.Pastorelli, N.Karro, M.Kalda

Annex 6
Essential publications 1999-2015

Books

A.Berezovski, J.Engelbrecht, G.A.Maugin. Numerical Simulation of Waves and Fronts in Homogeneous Solids. World Scientific, New Jersey et al., 2008.

T. Soomere, E.Quak (eds) Applied Wave Mathematics. Springer, Heidelberg, 2009.

J.Janno, J.Engelbrecht. Microstructured Materials: Inverse Problems. Springer, Heidelberg et al., 2011.

T. Soomere, E.Quak (eds) Preventive Methods for Coastal Prootection: Towards the Use of Ocean Dynamics for Pollution Control. Springer, Chan et al., 2013

J.Engelbrecht. Questions About Elastic Waves. Springer, Chan et al., 2015

Edited journals

Special issue on nonlinear dynamical phenomena.
Proc.Estonian Acad. Sci. Phys. Math., 1999, 48, 3-4

Special issue on nonlinear waves in solids. Eds G.A.Maugin, J.Engelbrecht, A.M. Samsonov
Wave Motion, 2001, 34, 1

Special issue on nonlinear waves in microstructured solids – Euromech 478. Eds. J.Engelbrecht, M.Kutser, G.A.Maugin
Proc.Estonian Acad. Sci. Phys. Math., 2003, 52, 1

Special issue on interaction phenomena in multiphase flows – Euromech 447. Eds J.Engelbrecht, Ü.Rudi
Proc. Estonian Acad. Sci. Engng, 2005, 11, 2

Special issue on systems biology of mitochondrian. Eds D.Beard, M.Vendelin
Am. J. Physiol. Cell Physiol., 2006, 291

Special issue on non-equilibrium dynamical phenomena in inhomogeneous solids – Euromech 478. Eds J.Engelbrecht, G.A.Maugin
Proc.Estonian Acad. Sci. Phys. Math., 2007, 56, 2

Special issue on oceanography, meteorology and coastal engineering. Eds T.Soomere, S.Keevallik.
Estonian J.Engng, 2007,13, 3

Special issue on oceanography, meteorology and coastal engineering. Eds T.Soomere, S.Keevallik.
Estonian J.Engng, 2008,14, 3

Special issue on continuum physics and engineering applications. Eds H.Herrmann, P.Van
Proc. Estonian Acad. Sci., 2008, 57,3

Special issue on oceanography, meteorology and coastal engineering. Eds. T.Soomere, K.E.Parnell, S.Keevallik.
Estonian J.Engng, 2009,15, 3

Special issue on complexity of nonlinear waves – Int. Conf, Tallinn.
Proc. Estonian Acad. Sci., 2010, 59, 2.

Special issue on advanced modelling of wave propagation in solids. Eds. J.Engelbrecht. D.Givoli, T. Hagstrom, G.A.Maugin
Wave Motion, 2013, 50, 7.

Special issue on complexity of nonlinear waves – IUTAM Symp. Eds A.Salupere, G.A.Maugin
Proc. Estonian Acad. Sci., 2015, 64, 3&3S

Textbooks and teaching materials

Ü.Lepik, J.Engelbrecht. Kaoseraamat. Eesti TA Kirjastus, Tallinn, 1999
The Book of Chaos (in Estonian)

T.Soomere. Environmental Modelling for Wave Dynamics. SESREMO course, 2011

T.Soomere. Coastal Processes and Environmental Management. SESREMO course, 2011

H.Aben. Photoelasticity of Glass. Compendium on CD ROM, 2010

J. Kalda. Materials for the International Physics Olympiad:
- kinematics htpp://www.ioc.ee/~kalda/ipho/kin_ENG.pdf
- mechanics htpp://www.ioc.ee/~kalda/ipho/meh_ENG.pdf
- circuits htpp://www.ioc.ee/~kalda/ipho/e-circuits.pdf
- wave optics htpp://www.ioc.ee/~kalda/ipho/waveopt.pdf
- formulas htpp://www.ioc.ee/~kalda/ipho/formulas.pdf

Essays

J.Engelbrecht. Mõtterajad. TA, Tallinn, 2004 (Roads of Thoughts – in Estonian)

J.Engelbrecht. Attractors of Thoughts. TA, Tallinn, 2004

J.Engelbrecht, N.Mann. The Sum of the Parts: ALLEA and Academies. ALLEA, Amsterdam, 2011

J.Engelbrecht. Mõtteraamat, Tallinn, 2014 (Collection of Thoughts – in Estonian)

Overviews in Estonian (incl articles)

M.Rahula. Matemaatika ilu ja võlu. Avita, Tartu, 2001
Beauty and Charm of Mathematics

T.Kändler, J.Engelbrecht (toim.) Keeruka maailma ilu. CENS, 2006
Beauty of Complex World

J.Engelbrecht, T.Kändler (toim). Keeruka maailma võlu. CENS, 2015
Charm of Complex World

J.Engelbrecht. Komplekssüsteemid meis ja meie umber. Horisont, 2014, No 2
Complex systems with and around us

T.Soomere. Märatsev meri – õppetunnid India ookeani tsunamist. Horisont, 2002, No2, 10-17
Furious sea – lessons from the Indian ocean tsunami

T.Soomere. Märatsev meri II – kui vesi ründab. Horisont, 2002, No 3, 32-38
Furious sea II – when water attacks

T.Soomere, J.Engelbrecht. Märatsev meri III - hiidlained. Horisont, 2006, No1, 28-33
Furious sea III – monster waves

J.Engelbrecht. Komplekssüsteemid. Akadeemia, 2010, No 8, 1347-1362
Complex systems

R.Kitt. Komplekssed sotsiaalsüsteemid. Akadeemia, 2011, No 10, 1787-1800
Complex social systems

Miscellaneous – history, prefaces, translations

Küberneetika Instituut muutuvas ajas. M.Kutser (toim), KübI, Tallinn, 2000
Institute of Cybernetics over changing time

Teadusmõte Küberneetika Instituudis. J.Engelbrecht, M.Kutser (toim), KübI, Tallinn, 2010
Scientific thoughts in the Institute of Cybernetics

J. Engelbrecht. Eessõna P.Milleri raamatule “Tark Parv”, Äripäev, 2011.
Preface to the translation of P.Miller “The Smart Swarm”.

T. Soomere. Tagasi laine ja aine partnerluse juurde. Eessõna G. Pretor-Pinney raamatule “Lainevaatleja käsiraamat”, Äripäev, 2012.

Preface to the translation of G.Pretor-Pinney "The Wavewatcher's Companion".
J.Engelbrecht. Eessõna T.Puu raamatule "Kunst, teadus, majandus", Swedbank, 2015.
Preface to the translation of T.Puu "Arts, Sciences and Economics"
M. Rahula (tõlkija), H. Poincaré "Viimased mõtted", Akadeemia, 8-11, 2006; 1,2, 2007.
Translation of H.Poincaré " Dernièrs pensées".

Bibliographies

U.Nigul (Ed). Deformation waves and acoustic evaluation. Institute of Cybernetics, Academy of Sciences, 1982 (covers 1960-1982). ??????
M. Kutser. Mechanics at the Institute of Cybernetics. Proc. Estonian Acad. Sci. Eng, 2000, 6, 3, 230-251 (selected publications over 1960-2000).
CENS Selected bibliography on waves in microstructured solids. Research Report Mech 308/14, Institute of Cybernetics (covers 1983-2014)
J.Engelbrecht. Wave propagation. Overview on studies 1983-2015. Research Report Mech 314/15, Institute of Cybernetics (annotated bibliography).
CENS Bibliography 2011-2015. Research Report Mech 314/15, Institute of Cybernetics
CENS Bibliography 1999-2010. Research Report Mech 315/15, Institute of Cybernetics

Annex 7

Selected invited lectures

J.Engelbrecht. Lagrangian lecture "Complexity in mechanics", University of Turin, Torino, May, 2009
J.Engelbrecht. Harold J. Gay lecture "Modeling of deformation waves in microstructured materials", Worcester Polytechnic Institute (WPI), Worcester, Jan., 2013
H.Aben. William Murray lecture " Photoelastic tomography with linear and nonlinear algorithms", Annual Conference of the Society for Experimental Mechanics, Indianapolis, June 2010.
T.Soomere. Invited public lecture "The smart use of currents for environmental management of marine activities", James Cook University, Townsville, Australia, Sept., 2015.
Alumäe lectures, Estonian National Committee for Mechanics:

T.Soomere (2000)
J.Engelbrecht (2002)
H.Aben (2004)
J.Kalda (2008)
M.Vendelin (2012)

Annex 8
Awards

State decorations
J.Engelbrecht: Coat of Arms IV Class,1999; III Class, 2007
H.Aben: White Star III Class, 2001
L.Ainola: White Star III Class, 2003
A.Salupere: White Star IV Class, 2013
J.Kalda: White Star V Class, 2013
T.Soomere: White Star III Class, 2014

State research awards
H.Hinrikus – 2000 in technical sciences
T.Soomere, J. Elken, T.Kõuts, J.Kask, U.Liiv – 2002 in technical sciences
A.Berezovski – 2004 in technical sciences
V.Saks, J.Engelbrecht, E.Seppet, M.Vendelin – 2008 in biosciences
H.Aben, L.Ainola, J.Anton, A. Errapart – 2009 for outstanding innovative research
J.Janno – 2012 in exact sciences
T.Soomere – 2013 in technical sciences
J.Engelbrecht – 2015 for life-long research

Other research awards
J.Engelbrecht – Dr.h.c. from the Budapest Technical University, 1999
J.Engelbrecht – Sign of Merit from the Estonian Ministry of Education and Research, 2004
J.Engelbrecht – Alumäe medal for research in wave mechanics, Estonian Acad Sci, 2005
T.Soomere – Baltic Assembly Science Prize, 2007
H.Aben – Alumäe medal for research in photoelasticity, Estonian Acad Sci, 2009
I.Didenkulova – Plinius medal for research in oceanography, European Gepophys Soc, 2010
H.Aben – Murray medal from Society for Experimental Stress Analysis, 2010
P.Saari – Denisyuk medal from Russian Optical Society for research in optics, 2011
T.Soomere – Medal of Baltic Academies for research in marine sciences,2013
J.Kalda – State President award for teaching physics, 2013

Annex 9
Geography of visitors

Amsterdam
Budapest, Bratislava, Blekinge, Bergen, Bologna, Berlin, Bialystok, Brest, Bucharest, Brasov, Bern, Brisbane
Cambridge, Chemnitz, Caen, Catania
Durham(US), Duisburg, Dundee, Delft, Dublin
Exeter, Enschede, Edinburgh, Eindhoven
Freiburg
Graz, Gdansk, Göttingen, Geestacht, Greifswald, Grenoble
Hanover (US), Helsinki, Hong Kong, Haifa, Houston
Kharkov, Kyoto
Linköping, London, Leuven, Loughborough, Lund, Limoges, Linz
Madrid, Moscow, Munich, Mexico, Marseille
Nizhny-Novgorod, Norwich, Nantes
Oslo, Osaka, Oxford, Oulu
Paris, Prague, Pusan, Patras, Palermo, Pretoria
Rostock, Riga, Rouen, Regensburg
Stockholm, Saarbrücken, Sofia, Seattle, St Petersburg, Stuttgart, Sopot, Salento
Torino, Tampere, Trondheim, Townsville, Tokyo, Turku
Umea, Ulm
Venice, Vilnius, Vladivostok
Warsaw, Waikato, Worcester, Wroclav, Warnemünde, Wuppertal
Zürich
York, Yokohama

Annex 10
Geography of visits

UK and Ireland
London, Edinburgh, Dublin, Brighton, Stratford on Avon, Oxford, Cambridge, Plymouth, Galway, Loughborough, Coventry, Cardiff, Birmingham, Sheffield, Aberdeen, Swansea, Warwick, Belfast, Manchester
France
Paris, Metz, Strasbourg, Poitiers, Valenciennes, Nice, Marseille, Montpellier, Toulouse, Lyon, Grenoble, Bordeaux, Brest, Aussois, Archachon, Porquerolles, Nancy, Rouen, Angers, Le Mans, Antibes
Spain and Portugal
Madrid, Barcelona, Lisbon, Porto, Santiago de Compostela, Sevilla, Bilbao, Las Palmas, Tenerife, Aveiro, Ponta Delgade
Nordic countries
Oslo, Copenhagen, Aalborg, Stockholm, Karlskrona, Norrköping, Helsinki, Lahti, Espoo, Tampere, Sigtuna, Göteborg, Lund, Uppsala, Lyngby, Roskilde, Reykjavik, Jyvaskyla, Umeå, Bergen, Lappenranta

Benelux

Brussels, Amsterdam, Brugge, Leuven, Delft, Eindhoven, Enschede, Liege, Groningen

Germany, Austria and Switzerland

Berlin, Heidelberg, Göttingen, Stuttgart, Duisburg, Darmstadt, Munich, Vienna, Graz, Warnemünde, Greifswald, Kaiserslautern, Aachen, Linz, Rostock, Chemnitz, Potsdam, Hannover, Saarbrücken, Dresden, Halle/Saale, Geestacht, Erlangen, Konstanz, Salzburg, Hamburg, Klagenfurt, Lausanne, Davos, Zürich, Düsseldorf, Magdeburg, Bern

Italy

Rome, Turin, Pavia, Como, Siena, Perugia, Bari, Bologna, Messina, Florence, Sorrento, Ragusa, Cortona, Venice, Naples, Ischia, Acireale, Avellino, Trento, Genova, Ferrara, Palermo, Milan, Pescara, Pisa, Cisterna di Latina, Lecce, Parma

North Eastern Europe

Warsaw, Krakow, Budapest, Riga, Kaunas, Torun, Prague, Gdansk, Klaipeda, Trakai, Sopot, Olomouc, Plovdiv, Bialystok, Szczecin, Vilnius, Palanga, Štrbské Pleso, Miskolc, Lagow

South Eastern Europe

Thessaloniki, Ljubljana, Rovinj, Kiev, Odessa, Athens, Patras, Sofia, Skopje, Istanbul, Rhodes, Crete, Aleksandropolis, Sibiu, Larissa, Dubrovnik

Russia

Moscow, St Petersburg, Nizhni-Novgorod, Novosibirsk, Baltiisk, Kazan

Asia

Kyoto, Tokyo, Yonezawa, Nara, Nagoya, Singapore, Seoul, Bejing, Hong Kong, Bandung, Taipei, Bangalore, New Delhi, Hangzhou, Xian, Hanoi, Chongqing, Shanghai, Qingdao, Quangzhou, Bangkok, Hyderabad, Osaka, Ashgabat, Baku, Alma-Ata

Australia and New Zealand

Adelaide, Sydney, Queensland, Christchurch, Perth, Hamilton, Auckland, Wellington, Cairns, Brisbane, Townsville

USA and Canada

Ann Arbor, Athens (Georgia), Chicago, San Francisco, Worcester, New Orleans, Boulder, Atlanta, Quebec, San Diego, Blacksburg, Orlando, Miami, Philadelphia, Salt Lake City, Baltimore, New London, Indianopolis, Havai, Washington, Seattle, Long Beach, Boston, Champagne, Vancouver, Stanford, San Jose, Monterey, Montreal, Las Vegas, Memphis, Halifax

Others

Eilat, Cancun, Marrakech, Cairo, Santiago, Puebla, Doha, Cape Town

Annex 11
Essential meetings organized

International Glass Stress Summer Schools, annually 2001-2016, Tallinn
EUROMECH Colloquium 436. Nonlinear Waves in Microstructured Solids. May 29-June 01. 2002, Tallinn
Advanced Study School. Nonlinear Processes in Marine Sciences, Oct, 2003, Hageri (Estonia)
International Workshop on Biomedical Engineering. Oct 2004, Tallinn.
EUROMECH Colloquium 478. Non-equilibrium Dynamical Phenomena in Inhomogeneous Solids. June 13-16, 2006, Tallinn.
Summer School on Applications of 3D Shapes. July 19-25, Tallinn
French-Estonian-Russian ECO-NET Seminar on Wave Current Interaction in Coastal Environment. Dec 7-8, 2006, Tallinn.
International Summer School. Waves and Coastal Processes. Aug 25- Sept 09, 2007, Tallinn.
International Conference on Complexity of Nonlinear Waves. Oct 5-7, 2009, Tallinn.
International Seminar Complex Systems: growth and emergent behaviour. Oct 9, 2009, Tallinn.
International Summer School: Scientific Computing. Aug 7-11, 2010, Tallinn
Humboldt Colloquium, Sept 02-04, 2010,Tallinn.
Meeting of the Academic Council of the State President. Complexity, March 24, 2011, Tallinn.
The 2nd Baltic Way Annual meeting. April 11-13, 2011, Palermo, Italy.
International Summer School: Preventive Methods foe Cioastal Protection. Sept 18-20, Klaipeda, Lithuania
The 10th Eurographics Symposium for Geometry Processing. July 16-18, 2012, Tallinn
International Workshop. Measurement, Visualisation, Modelling and Simulation of Complex/Microstructured Materials, June 5-6, 2012, Tallinn
EUROMECH Colloquium 540. Advanced Modelling of Wave propagation in Solids, Oct 1-3, 2012, Prague, Czech Republic.
IUTAM Symposium on Complexity of Nonlinear Waves. Sept 8-12, 2014, Tallinn.
InternationalTraining Event, Tempus SESREMO project on Coastal Environment, Nov 1-14, 2014, Klaipeda, Lithuania.
The 28th Nordic Seminar on Computational Mechanics, Oct 22-23, 2015, Tallinn.

Annex 12
Essential international grants

BMBF (Germany) grant 03 No 9005/2. Nonlinear dynamics of heterogeneous solids with microstructure (partner: RWTH Aachen).
ESF grant NATEMIS. Nonlinear acoustic technique for micro-scale damage diagnostics.
NATO Collaborative Linkage grant. Thermomechanics of progress and stability of phase interfaces (partners: Paris 6, TU Berlin).
INSERM (France) grant. Modélisation biomathématique pour l'étude du métabolisme énergetique (partners: Grenoble, Bordeaux).
PAPA (EU) contract. Programme for a Baltic network to assess and upgrade an operational observing and forecasting system in the region.
WIND-CHIME (EU FP6 INCO project). Wide-range non-intrusive devices toward conservation of historical monuments in the Mediterranean area.
CENS-CMA (Marie Curie ToK). Cooperation of Estonian and Norwegian Scientific Centres within Mathematics and its Applications (partners: CENS and Centre for Mathematics and its Applications, University of Oslo).
ERA-NET Complexity (FP 6).
SEAMOCS (Marie Curie Research and Training Network). Applied stochastic models for ocean engineering, climate and safe transportation.
AIM@SHAPE (FP6 EU IST Network of Excellence on shape modelling).
Wellcome Trust International Senior Research Fellowship (M.Vendelin). Analysis of structural and functional aspects of compartmentation of adenine nucleotides in heart muscle cells.
Humboldt Foundation Feodor Lynen Fellowship (H.Herrmann).
FP7 FET, Global System Dynamics and Policies: simulation and visualisation technologies.
BONUS+ project Baltic Way. The potential of currents for environmental management of the Baltic Sea maritime industry.
Marie Curie re-integration grant ESTSpline (E.Quak).
FP7 ESTwave. Educational, scientific and technological aspects of mesoscopic continuum physics for waves in complex materials. (H.Herrmann).
FP7 FET Pilot project FuturICT.
MOBILITAS Top Researcher Grant (T.Torsvik).